Graph Search

CMPUT 261: Introduction to Artificial Intelligence

P&M §3.1-3.4

Recap: Course Essentials

Course information: https://jrwright.info/introai/

- This is the main source of information about the class
- Syllabus, slides, readings, deadlines

Lectures: Tuesdays and Thursdays,12:30-1:50pm in **ED 2-115**

- In person

eClass: https://eclass.srv.ualberta.ca/course/view.php?id=95479

- Discussion forum for **public** questions about assignments, lecture material, etc.
- Handing in assignments

Email: james.wright@ualberta.ca for **private** questions

- (health problems, inquiries about grades)

Office hours: By appointment, or after lecture

- TA's are available to help during lab hours
- Labs begin **next week**; they will include a brief Python tutorial

Recap: Search

Example: Farmer's raft

A farmer needs to move a hen, fox, and bushel of grain from the left side of the river to the right using a raft.

- The farmer can take one item at a time (hen, fox, or bushel of grain) using the raft.
- The hen cannot be left alone with the grain, or it will eat the grain.
- The fox cannot be left alone with the hen, or it will eat the hen.

- We want to compute a sequence of actions:
  - from a **starting state** (all of the animals on the left bank)
  - to a **goal state** (all of the animals on the right bank)
  - while satisfying **constraints** (nothing gets eaten)
- Every action has a **known** and **deterministic** result and cost
- **Search:** efficiently compute a cost-optimal solution based on known rules

Lecture Outline

1. Recap & Logistics
2. Search Problems
3. Graph Search
4. Markov Assumption

After this lecture, you should be able to:

- Represent a search problem formally
- Represent a search problem as a search graph
- Implement a generic graph search
- Identify whether a representation satisfies the Markov assumption

Search

- It is often easier to **recognize** a solution than to **compute** it
  - Search exploits this property!
- Agent searches **internal representation** to find solution
  - Outcomes are **known** and **deterministic**, so no need for observations
  - All computation is purely internal to the agent.
- Formally represent as searching a **directed graph** for a path to a goal state
- **Question:** Why might this be a good idea?
  - Because it is very **general**. Many AI problems can be represented in this form, and the same algorithms can solve them all.

State Space

- A **state** describes all the relevant information about a possible configuration of the environment
- **Markov assumption**: How the environment got to a given configuration doesn't matter, just the current configuration.
  - It is always possible to construct such a representation (**how?**)
- A state is an assignment of values to one or more **variables**, e.g.:
  - A single variable called "state"
  - x and y coordinates, temperature, battery charge, etc.
- **Actions** change the environment from one state to another

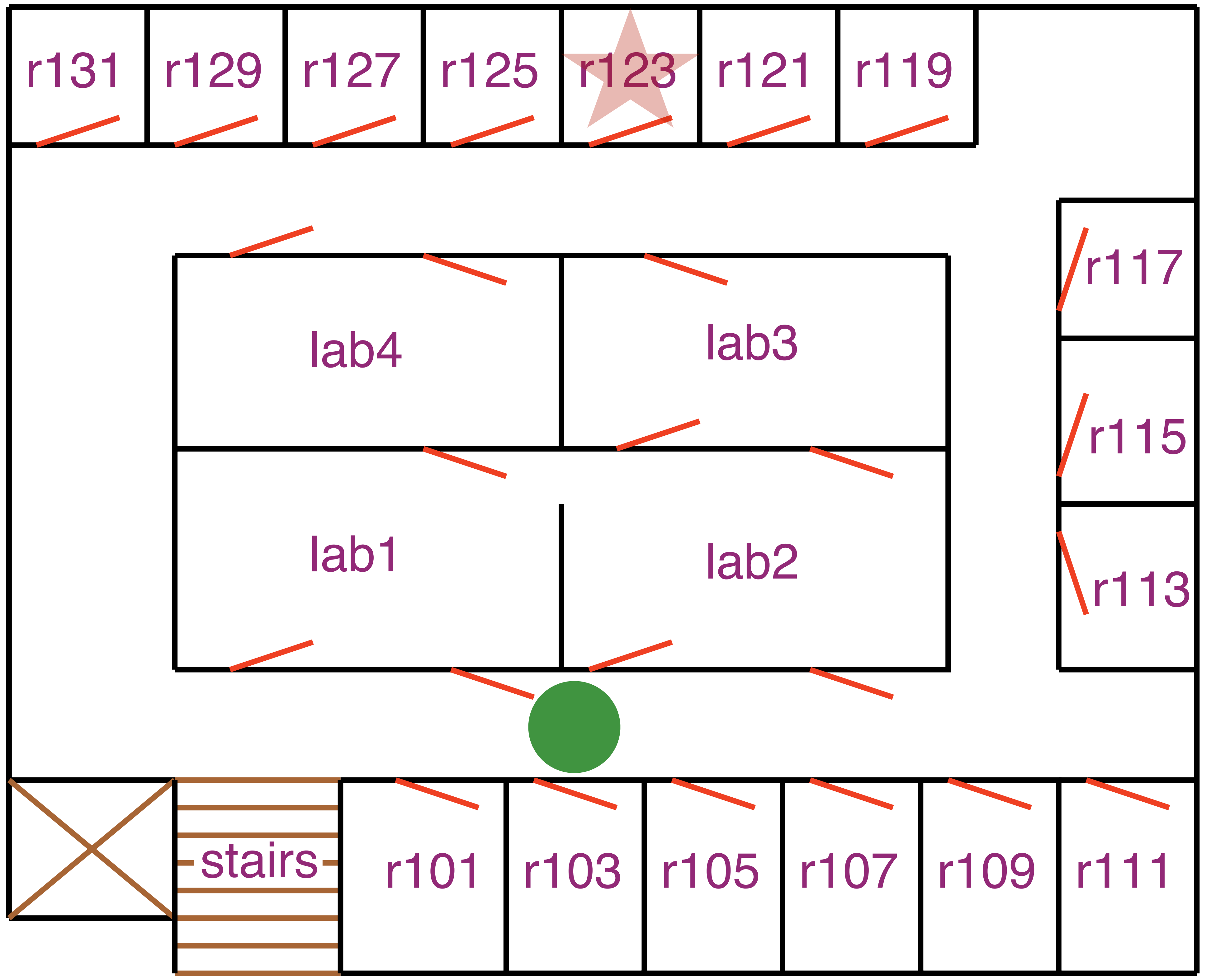
Search Problem

Definition: Search problem **(textbook: state-space problem)**

- A set of **states**
- A **start state** (or set of start states)
- A set of **actions** available at each state
- A **successor function** that maps from a state to a set of reachable states
    - The textbook calls this an "action function"
- A **cost** for moving from each state to each successor state
- A **goal function** that returns true when a state satisfies the goal

Example: DeliveryBot

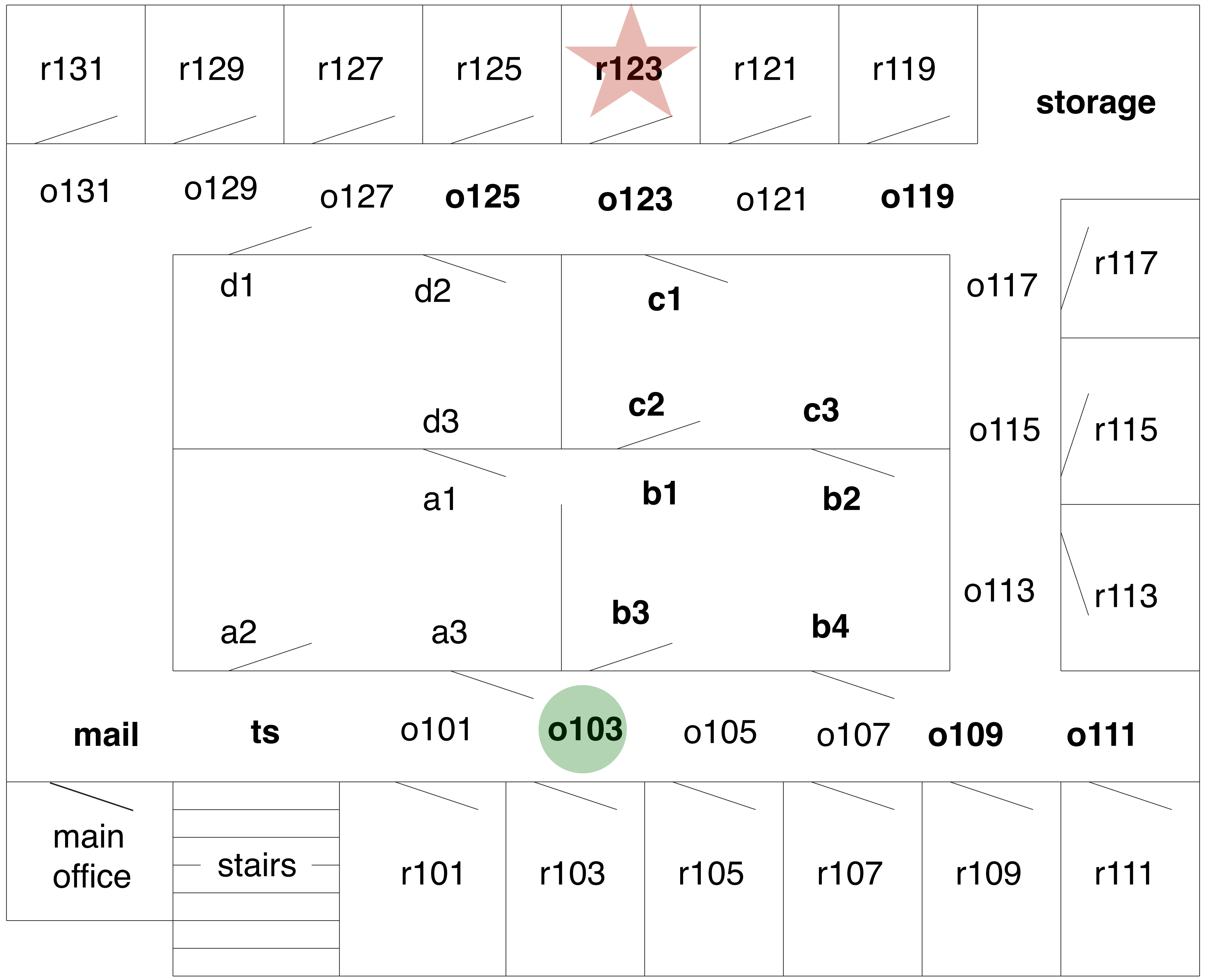
DeliveryBot wants to get from outside room 103 to inside room 123



Question: What might be a better representation for states?

DeliveryBot as a Search Problem

States	{r131, o131, r129, o129, ...}
Actions	{go-north, go-south, go-east, go-west}
Start state	o103
Successor function	succ(r101) = {r101, o101}, succ(o101) = {o101, lab1, r101,o105, ts}, ...
Goal function	$\text{goal}(s) = \begin{cases} 1 & \text{if } s = \text{r123}, \\ 0 & \text{otherwise.} \end{cases}$



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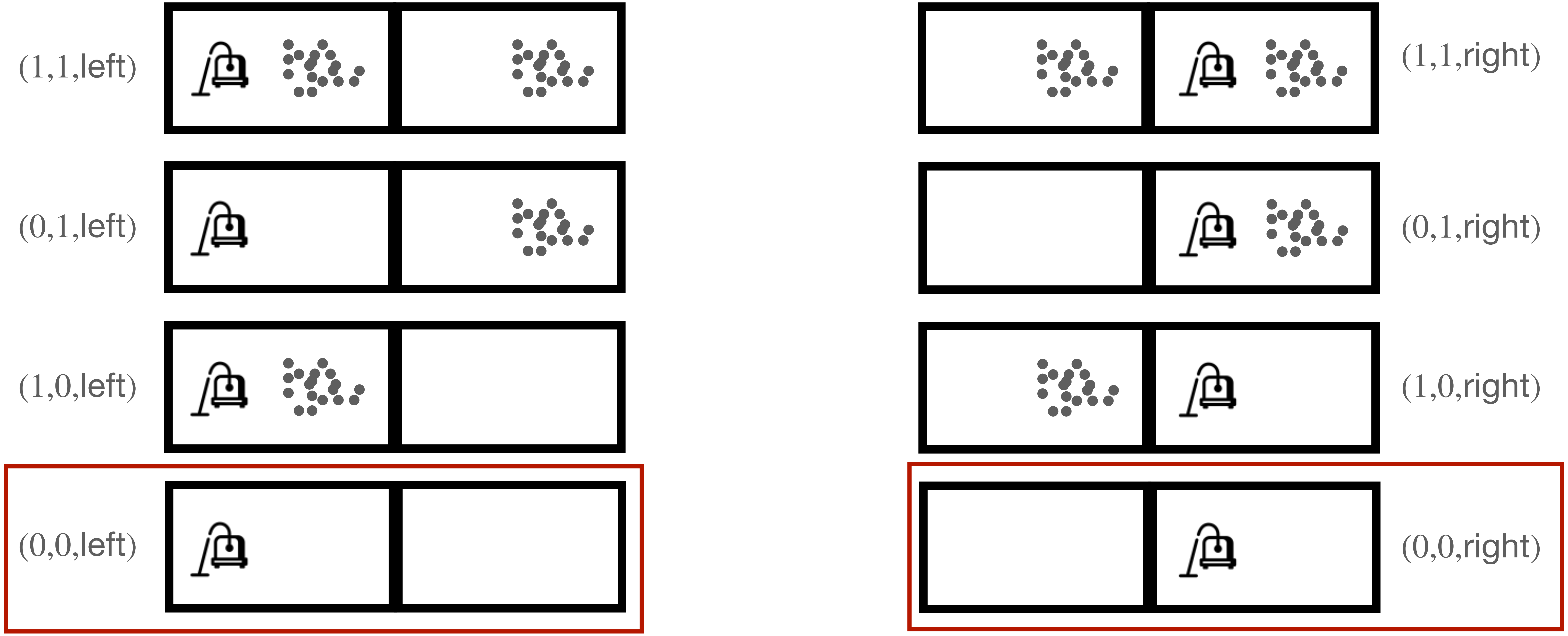
Example: VacuumBot

- Two rooms, one cleaning robot
- Each room can be clean or dirty
- Robot has two actions:
  - **clean**: makes the room the robot is in clean
  - **move**: moves to the other room
- Robot's goal: All the rooms are clean

Questions:

1. How many **states** are there
2. How many **goal states**?

VacuumBot as a Search Problem: States



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Solving Search Problems, informally

1. Consider each **start state**

2. Consider every state that can be **reached** from some state that has been previously considered (and remember how to reach the state)

3. **Stop** when you encounter a **goal state**, output plan for reaching the state

Directed Graphs

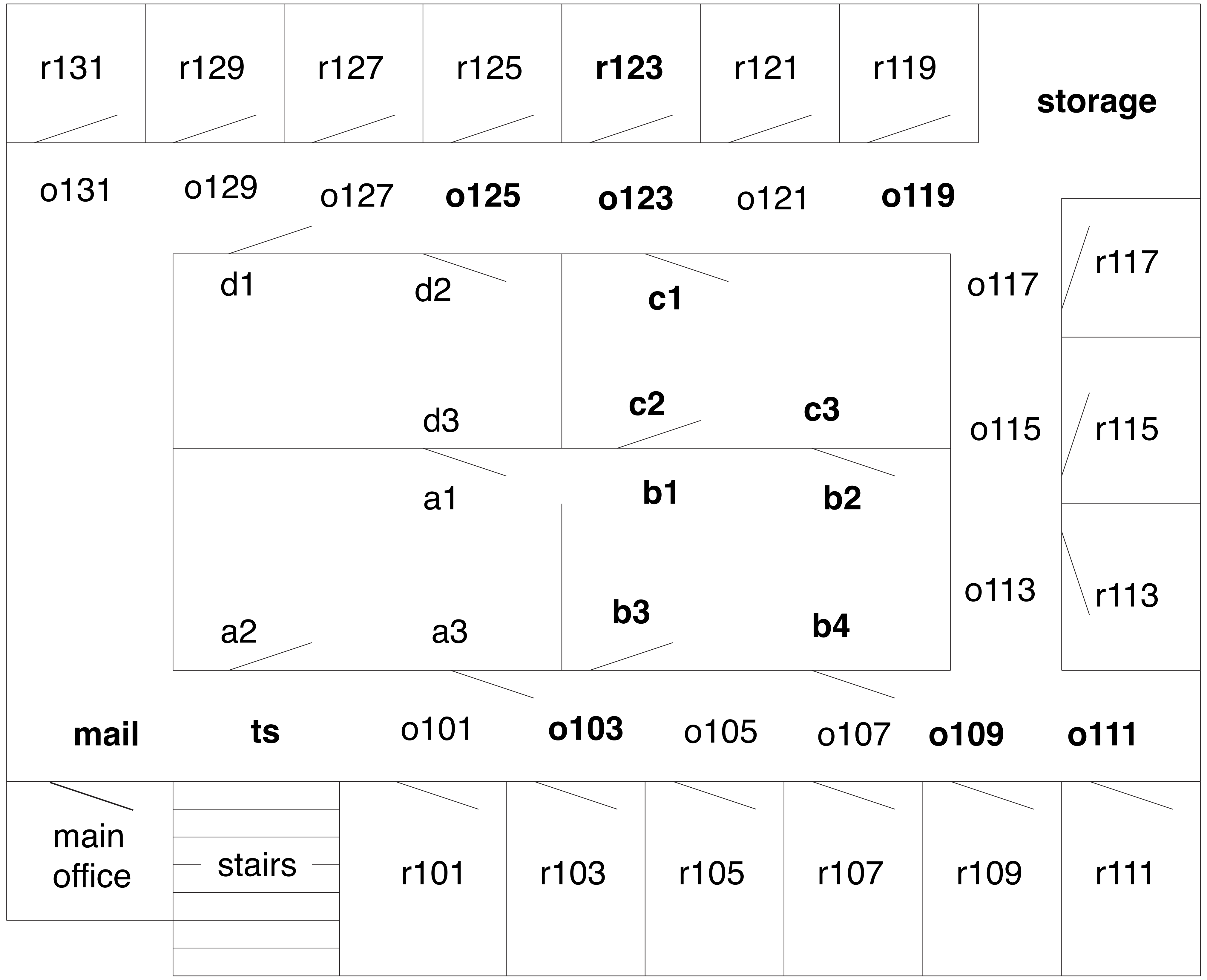
- A **directed graph** is a pair $G = (N, A)$
  - N is a set of **nodes**
  - A is a set of ordered pairs called **arcs**
- Node n_2 is a **neighbour** of n_1 if there is an arc from n_1 to n_2
  - i.e., $\langle n_1, n_2 \rangle \in A$
- A **path** is a sequence of nodes $\langle n_0, n_1, \ldots, n_k \rangle$ with $\langle n_{i-1}, n_i \rangle \in A$
  - **Length** of a path is number of **arcs** (not nodes)

Search Graph

We can represent any search problem as a **search graph**:

1. Nodes are the **states**
2. Neighbours are the **successors** of a state
   - i.e., add one **arc** from state s to each of s's **successors**
3. A **solution** is a path $\langle n_0, n_1, \ldots, n_k \rangle$ from a **start node** to a **goal node**
4. Label each arc with the **cost** for transitioning to the successor state
5. *Optional:* Label each arc with the **action** that leads to the successor state
   - **Question:** Why is this optional?

DeliveryBot: Search Graph



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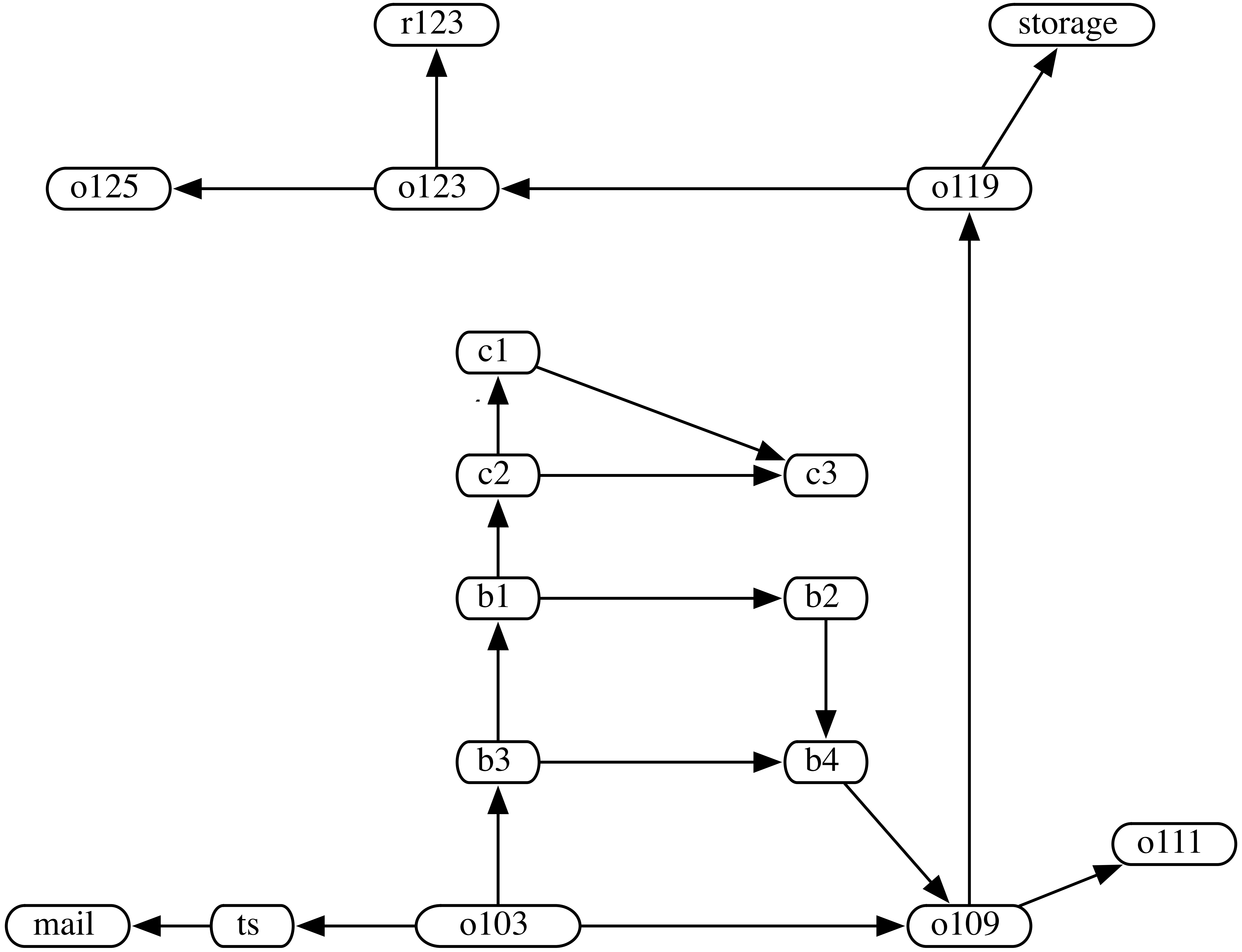

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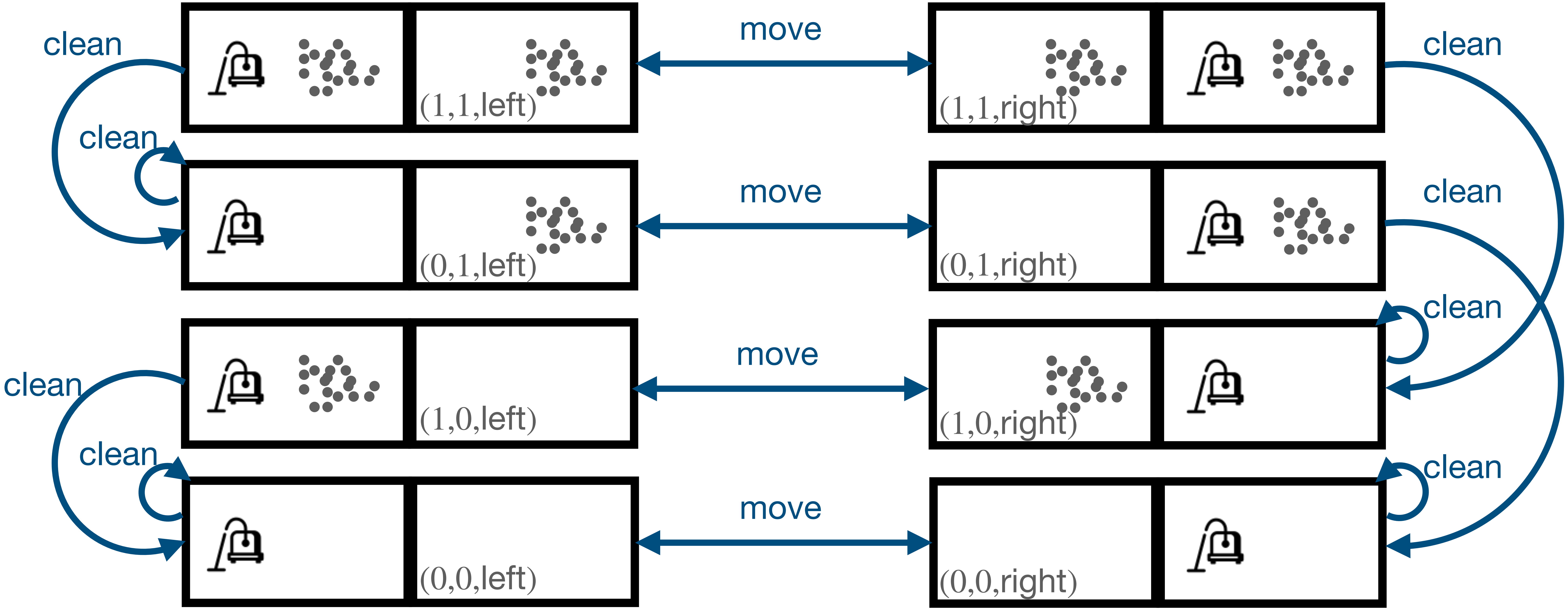
VacuumBot: Search Graph



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VacuumBot: Search Graph

$V = \{(0,0,\text{left}), (0,1,\text{left}), (1,0,\text{left}), (1,1,\text{left}), (0,0,\text{right}), (0,1,\text{right}), (1,0,\text{right}), (1,1,\text{right})\}$

$$A = \{\langle (x, y, p), (x', y', p') \rangle \mid (x', y', p') = f(x, y, p) \vee (x', y', p') = g(x, y, p)\}$$

$$f(x, y, p) = \begin{cases} (0, y, p) & \text{if } p = \text{left} \\ (x, 0, p) & \text{if } p = \text{right} \end{cases}$$

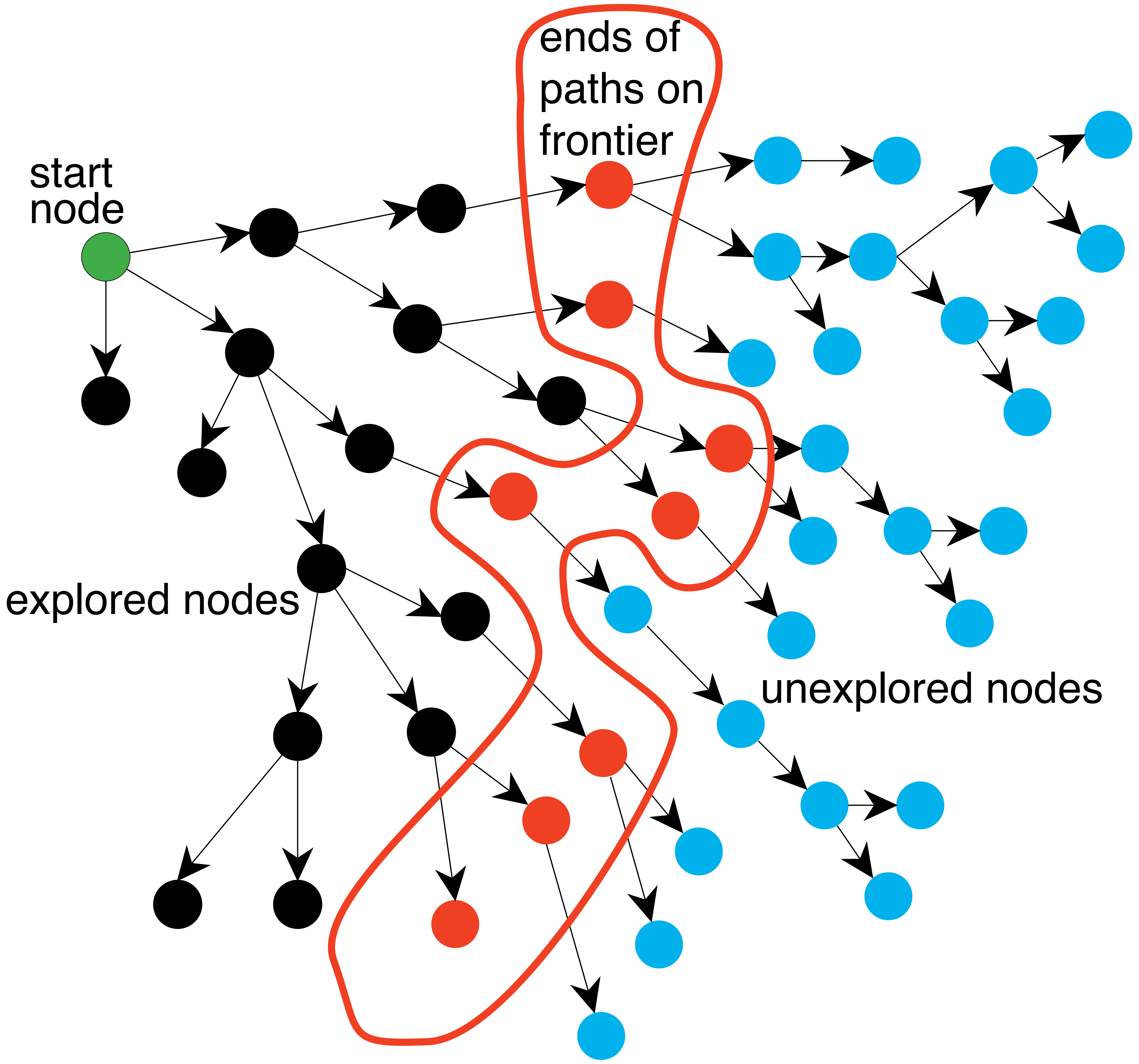
$$g(x, y, p) = \begin{cases} (x, y, \text{right}) & \text{if } p = \text{left} \\ (x, y, \text{left}) & \text{if } p = \text{right} \end{cases}$$

$$goal(x, y, p) = (x = 0 \wedge y = 0)$$

$$cost(v_1, v_2) = 1$$

Generic Graph Search Algorithm

- Given a graph, start nodes, and goal, incrementally explore paths from the start nodes
- Maintain a **frontier** of **paths** that have been explored
- As search proceeds, the frontier **expands** into the unexplored nodes until a goal is encountered.
- The **way** the frontier is expanded defines the **search strategy**



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Generic Graph Search Algorithm

Input: a *graph*; a set of *start nodes*; a $goal$ function

$frontier := \{\langle s \rangle \mid s \text{ is a start node}\}$
while $frontier$ is not empty:
    select a path $\langle n_0, \ldots, n_k \rangle$ from $frontier$
    remove $\langle n_0, \ldots, n_k \rangle$ from $frontier$
    if $goal(n_k)$:
        return $\langle n_0, \ldots, n_k \rangle$
    for each neighbour n of n_k:
        add $\langle n_0, \ldots, n_k, n \rangle$ to $frontier$
end while

Search Problem with Costs

What if solutions have differing qualities?

- Add **costs** to each arc: $\text{cost}(n_{i-1}, n_i)$
- **Cost of a solution** is the sum of the arc costs:

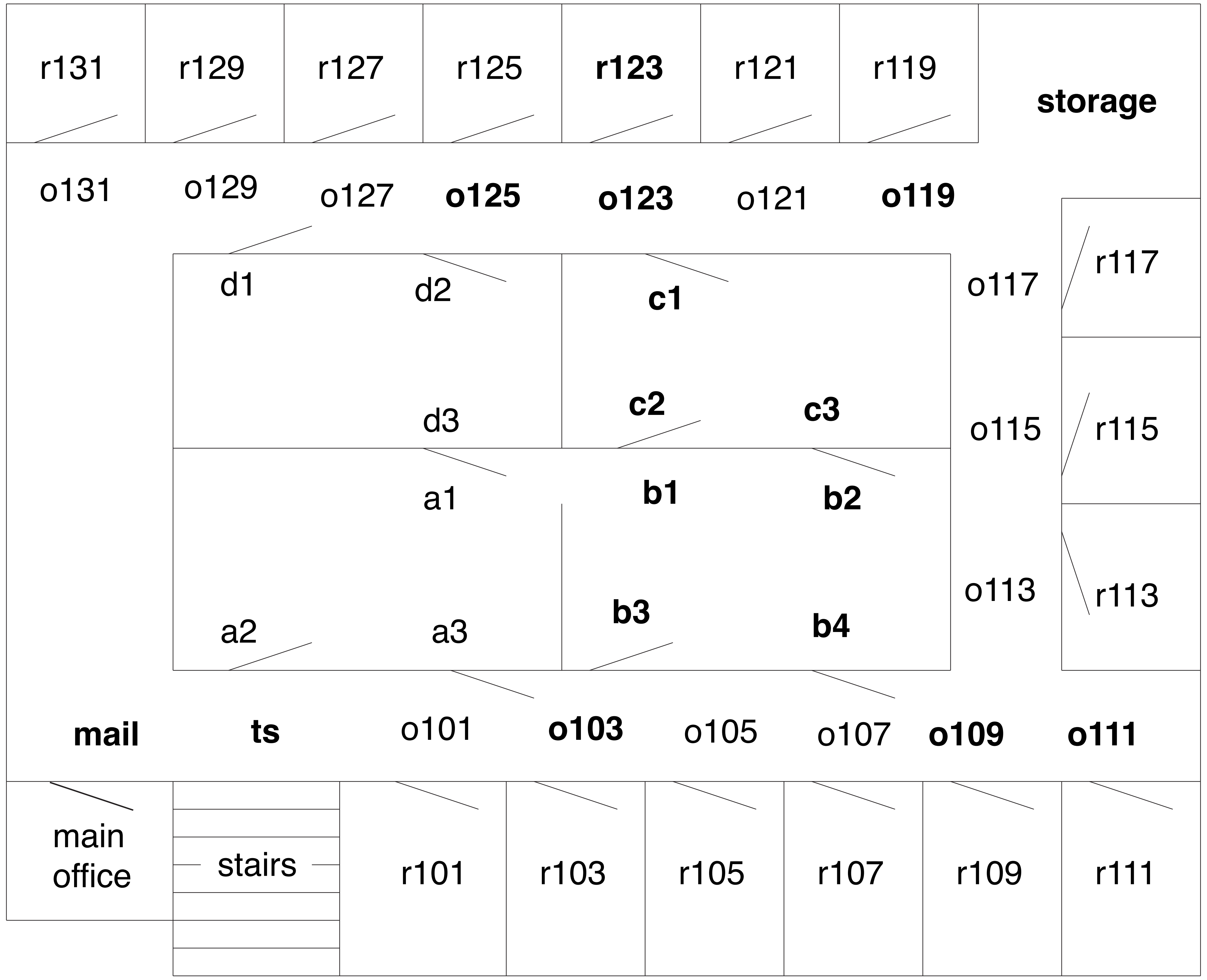
$$\text{cost}\left(\langle n_0, n_1, \ldots, n_k \rangle\right) = \sum_{i=1}^{k} \text{cost}(n_{i-1}, n_i)$$

- An **optimal solution** is one with the lowest cost

Questions:

1. Is this scheme sufficiently **general**?
2. What if we only care about the **number of actions** that the agent takes?
3. What if we only care about the **quality** of the end state (i.e., we don't care about the actions)?

DeliveryBot with Costs



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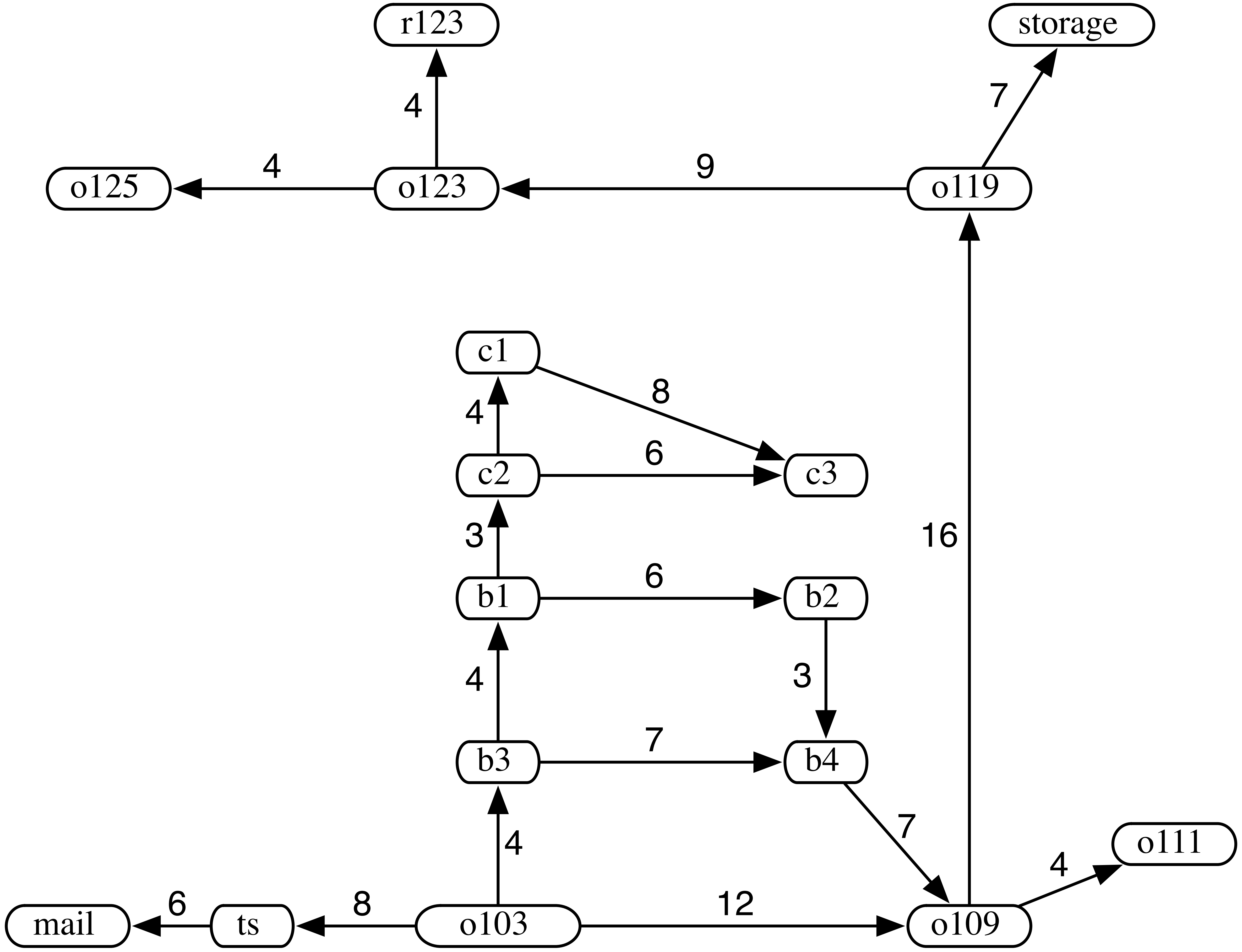

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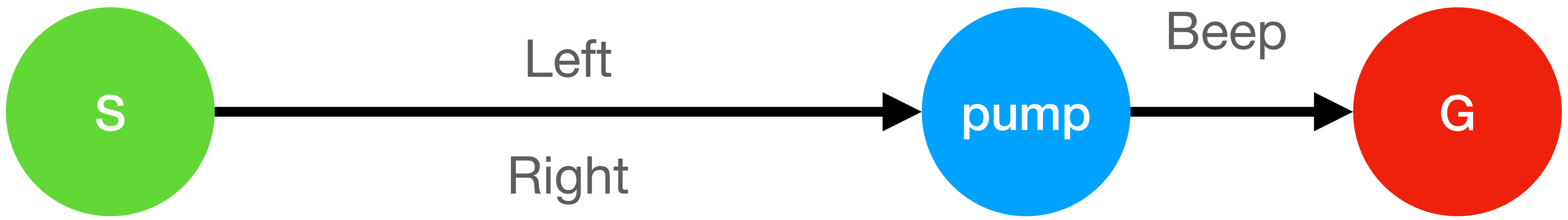
Markov Assumption

- *Informally:*
  How the environment arrived at the current configuration "doesn't matter"
- **Question:** What does "doesn't matter" mean *formally?*
- Edge costs, available actions, neighbourhoods, all depend only on **starting state** (and maybe action)
  - NOT on "sequence of edges that led to the current state"
- Mathematically, this means that each of these is a **function of** the **state** not the **history**
  - E.g., defining costs as $\text{cost}(s, z)$ instead of $\text{cost}(\langle n_0, n_1, n_2, s \rangle, z)$ **guarantees** that the representation satisfies the Markov assumption (with respect to costs)

Markov Assumption: GasBot

The **Markov assumption** is **crucial** to the graph search algorithm



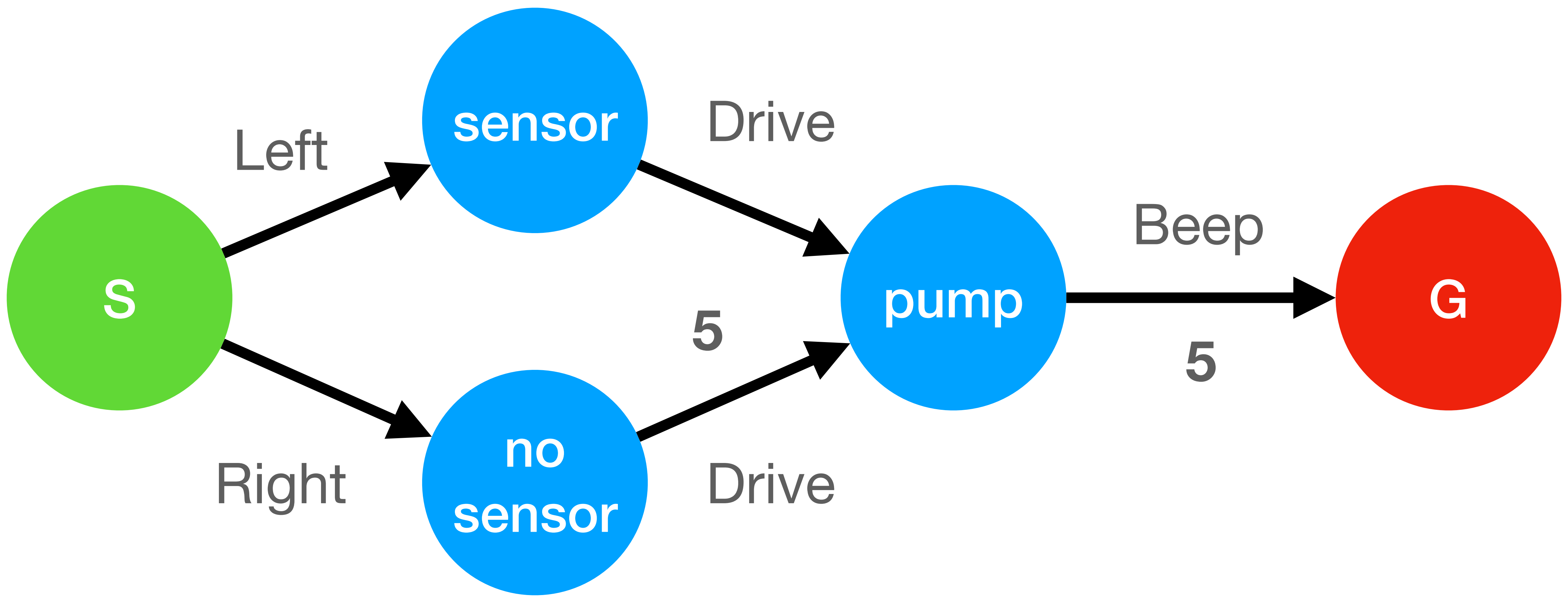
cost(pump, gas):
5 if went through sensor
10 otherwise

Getting to the pump:
from the **left** goes through sensor
from the **right** does not

Question: Does this representation representation satisfy the Markov assumption? Why or why not?

Markov Assumption: GasBot

The **Markov assumption** is **crucial** to the graph search algorithm



Questions

1. Does this representation satisfy the Markov assumption? Why or why not?
2. How else could we have fixed up the previous example?

Summary

- Many AI tasks can be represented as **search problems**
    - A single generic **graph search algorithm** can then solve them all!
- A search problem consists of **states**, **actions**, **start states**, a **successor function**, a **goal** function, optionally a **cost** function
- **Solution quality** can be represented by labelling arcs of the search graph with **costs**
- The **Markov assumption** is critical for graph search to work