Temporal Difference Learning

CMPUT 261: Introduction to Artificial Intelligence

S&B §6.0-6.2, §6.4-6.5

Lecture Overview

1. Recap & Logistics
2. TD Prediction
3. On-Policy TD Control (Sarsa)
4. Off-Policy TD Control (Q-Learning)
5. Expected Sarsa

After this lecture, you should be able to:

- trace an execution of the TD(0) algorithm
- trace an execution of the Q-learning algorithm
- trace an execution of the Sarsa algorithm
- define bootstrapping
- explain why bootstrapping is useful
- trace an execution of the Expected Sarsa algorithm
- describe the advantages of Expected Sarsa over Sarsa

Logistics

- **Assignment #4** is due **April 11** at 11:59pm
  - Late submissions for 20% deduction until **April 15** at 11:59pm
- **SPOT** (formerly USRI) surveys are now available
  - Available until **April 14**

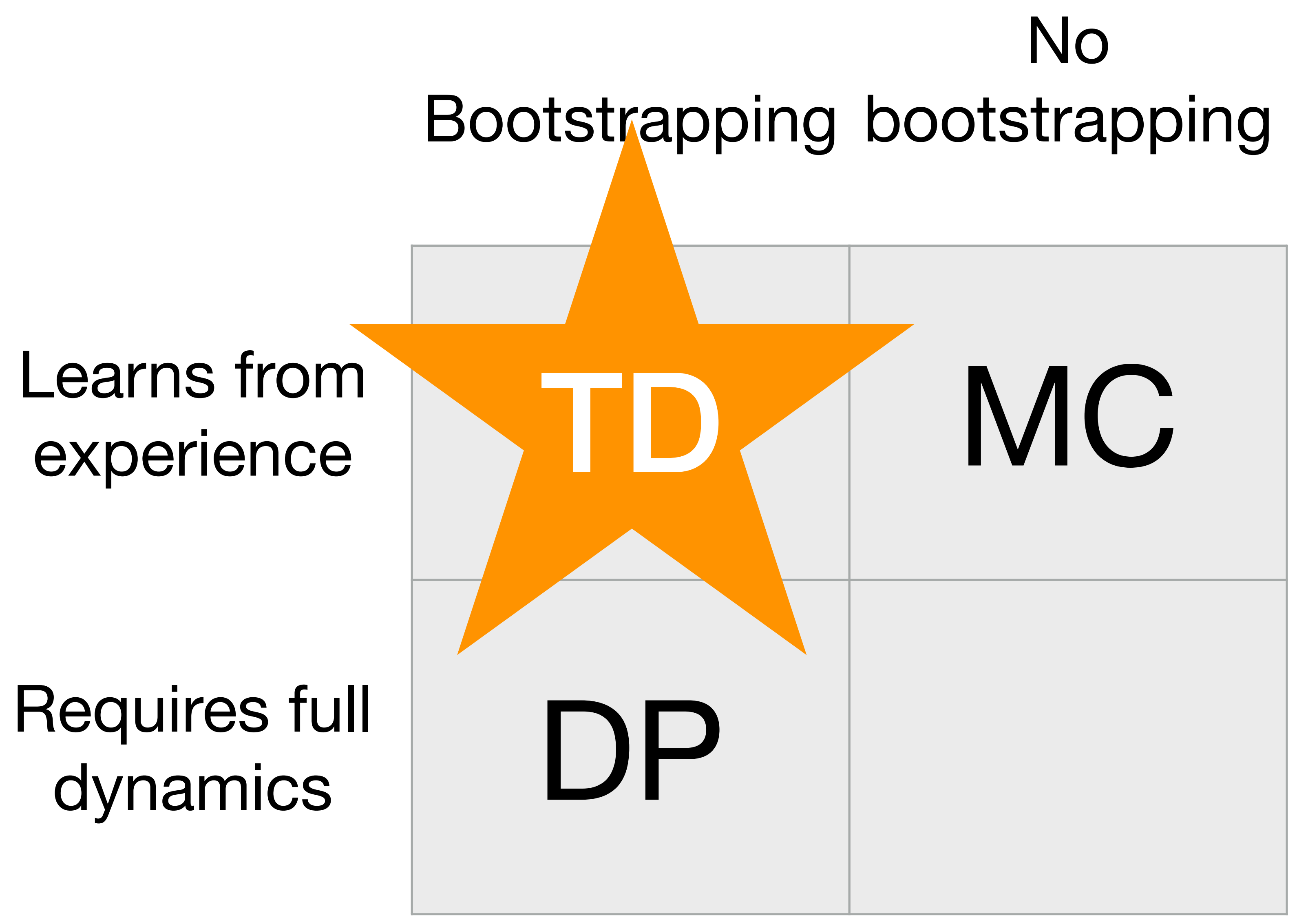
Previous Lecture Summary

- **Monte Carlo estimation** estimates values by averaging returns over **sample episodes**
  - Does not require access to full model of **dynamics**
  - Does require access to an entire **episode** for each sample
- Estimating **action values** requires either **exploring starts** or a **soft policy** (e.g., ϵ-greedy)
- **Off-policy learning** is the estimation of value functions for a **target policy** based on episodes generated by a different **behaviour policy**
  - **Importance sampling** is one way to perform off-policy learning
  - **Weighted** importance sampling has lower **variance** than **ordinary** importance sampling
- **Off-policy control** is learning the **optimal policy** (target policy) using episodes from a **behaviour policy**

Learning from Experience

- Suppose we are playing a blackjack-like game **in person**, but we **don't know the rules**.
  - We know the actions we can take, we can see the cards, and we get told when we win or lose
- **Question:** Could we compute an optimal policy using **dynamic programming** in this scenario?
- **Question:** Could we compute an optimal policy using **Monte Carlo**?
  - What would be the **pros and cons** of running Monte Carlo?

Bootstrapping



- Dynamic programming **bootstraps**: Each iteration's estimates are based partly on **estimates from previous iterations**
- Each Monte Carlo estimate is based only on **actual returns**

Updates

Dynamic Programming: $V(S_t) \leftarrow \sum_a \pi(a \mid S_t) \sum_{s',r} p(s', r \mid S_t, a)\left[r + \gamma V(s')\right]$

Monte Carlo: $V(S_t) \leftarrow V(S_t) + \alpha \left[G_t - V(S_t)\right]$

TD(0): $V(S_t) \leftarrow V(S_t) + \alpha \left[R_{t+1} + \gamma V(S_{t+1}) - V(S_t)\right]$

$$
\begin{aligned}
v_\pi(s) &\doteq \mathbb{E}_\pi[G_t \mid S_t = s] \\
&= \mathbb{E}_\pi[R_{t+1} + \gamma G_{t+1} \mid S_t = s] \\
&= \mathbb{E}_\pi[R_{t+1} + \gamma v_\pi(S_{t+1}) \mid S_t = s] .
\end{aligned}
$$

Monte Carlo: Approximate because of $\mathbb{E}$

Dynamic programming: Approximate because v_π not known

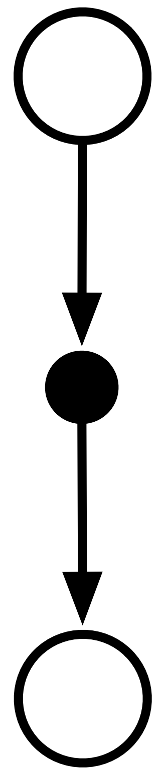
TD(0): Approximate because of $\mathbb{E}$ ***and*** v_π not known

TD(0) Algorithm

Tabular TD(0) for estimating v_π

Input: the policy π to be evaluated
Algorithm parameter: step size $\alpha \in (0, 1]$
Initialize $V(s)$, for all $s \in \mathcal{S}^+$, arbitrarily except that $V(\textit{terminal}) = 0$

Loop for each episode:
    Initialize S
    Loop for each step of episode:
        $A \leftarrow$ action given by π for S
        Take action A, observe R, S'
        $V(S) \leftarrow V(S) + \alpha\big[R + \gamma V(S') - V(S)\big]$
        $S \leftarrow S'$
    until S is terminal



Question: What information does this algorithm use?

TD for Control

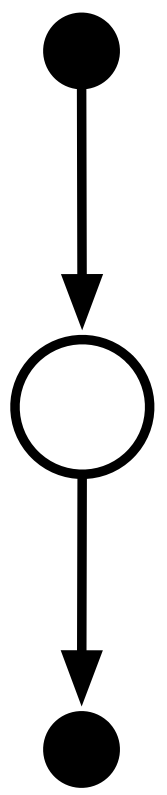
- We can plug TD prediction into the **generalized policy iteration** framework
- **Monte Carlo control loop:**
  1. Generate an **episode** using estimated π
  2. Update estimates of Q and π
- **On-policy TD control loop:**
  1. Take an **action** according to π
  2. Update estimates of Q and π

On-Policy TD Control

Sarsa (on-policy TD control) for estimating $Q \approx q_*$

Algorithm parameters: step size $\alpha \in (0, 1]$, small $\varepsilon > 0$
Initialize $Q(s, a)$, for all $s \in \mathcal{S}^+, a \in \mathcal{A}(s)$, arbitrarily except that $Q(\textit{terminal}, \cdot) = 0$

Loop for each episode:
  Initialize S
  Choose A from S using policy derived from Q (e.g., ε-greedy)
  Loop for each step of episode:
    Take action A, observe R, S'
    Choose A' from S' using policy derived from Q (e.g., ε-greedy)
    $Q(S, A) \leftarrow Q(S, A) + \alpha\big[R + \gamma Q(S', A') - Q(S, A)\big]$
    $S \leftarrow S'$; $A \leftarrow A'$;
  until S is terminal



Question: What **information** does this algorithm use?

Question: Will this estimate the Q-values of the **optimal** policy?

Actual Q-Values vs. Optimal Q-Values

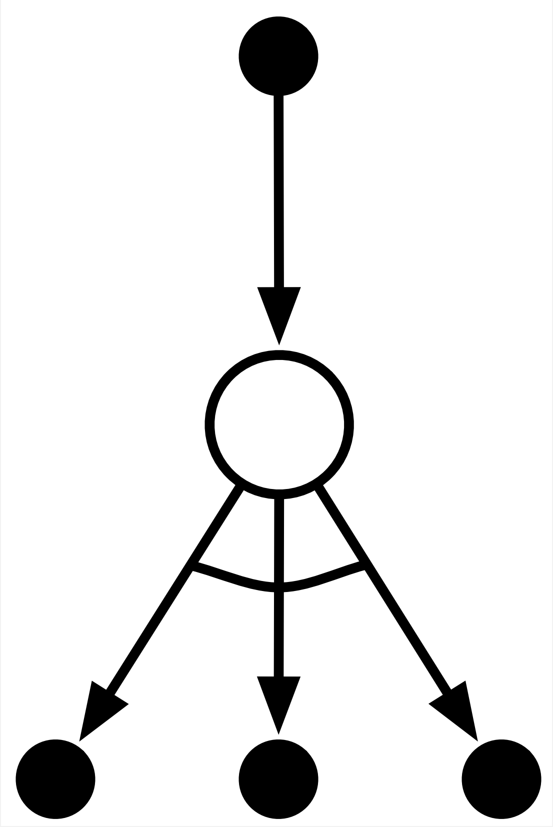
- Just as with on-policy Monte Carlo control, Sarsa does not converge to the **optimal** policy, because it always chooses an **ϵ-greedy action**
    - And the estimated Q-values are with respect to the **actual actions**, which are ϵ-greedy
- **Question:** Why is it necessary to choose ϵ-greedy actions?
- What if we **acted** ϵ-greedy, but **learned the Q-values** for the optimal policy?

Off-Policy TD Control

Q-learning (off-policy TD control) for estimating $\pi \approx \pi_*$

Algorithm parameters: step size $\alpha \in (0, 1]$, small $\varepsilon > 0$
Initialize $Q(s, a)$, for all $s \in \mathcal{S}^+, a \in \mathcal{A}(s)$, arbitrarily except that $Q(terminal, \cdot) = 0$

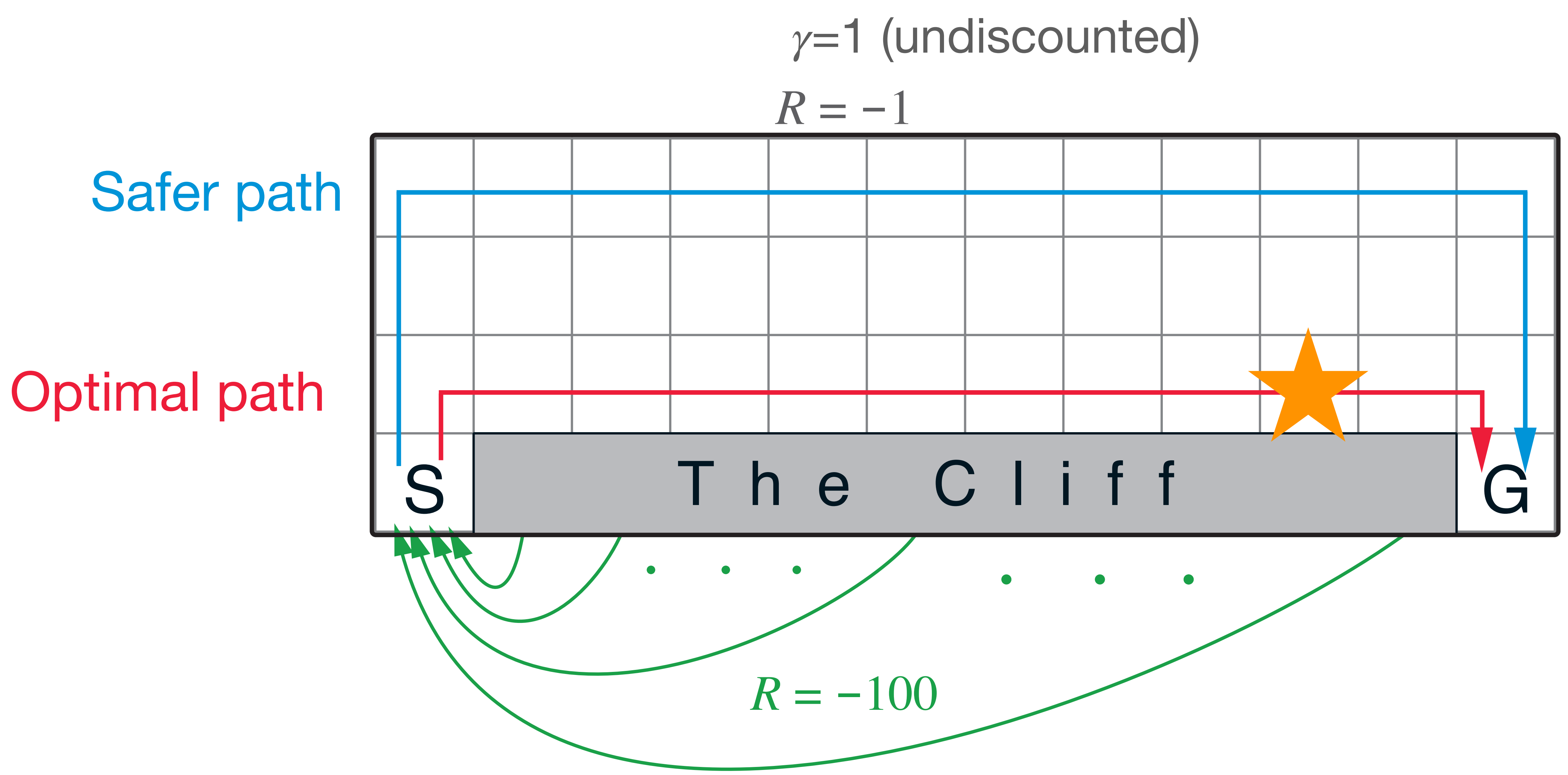
Loop for each episode:
  Initialize S
  Loop for each step of episode:
    Choose A from S using policy derived from Q (e.g., ε-greedy)
    Take action A, observe R, S'
    $Q(S, A) \leftarrow Q(S, A) + \alpha\big[R + \gamma \max_a Q(S', a) - Q(S, A)\big]$
    $S \leftarrow S'$
  until S is terminal



Question: What **information** does this algorithm use?

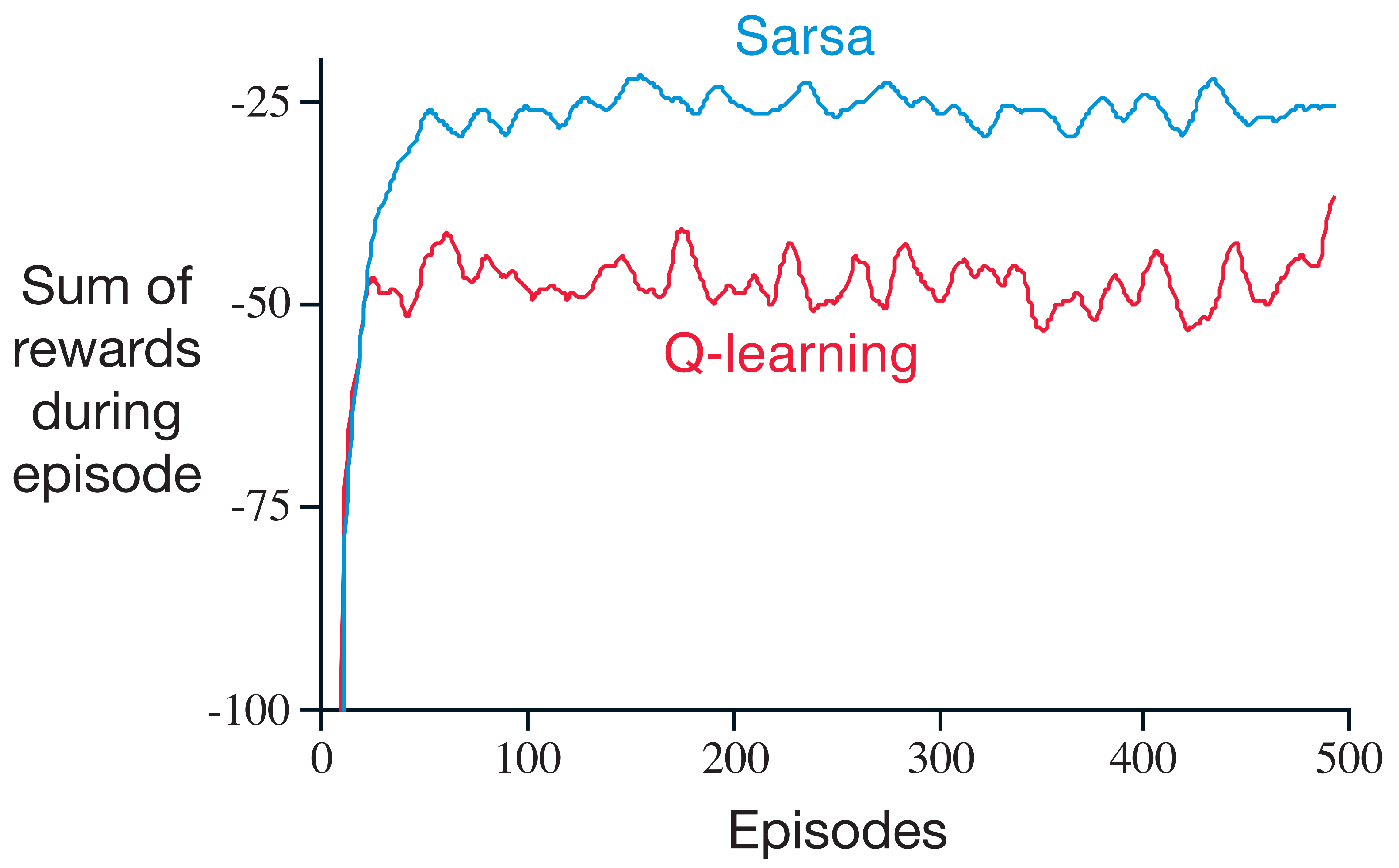
Question: Why aren't we estimating the **policy π** explicitly?

Example: The Cliff



- Agent gets -1 reward until they reach the goal state
- Step into the Cliff region, get reward -100 and go back to start
- **Question:** How will **Q-Learning** estimate the value of **this** state?
- **Question:** How will **Sarsa** estimate the value of **this** state?

Performance on The Cliff



Q-Learning estimates **optimal policy**, but Sarsa consistently **outperforms** Q-Learning. (**why?**)

Sarsa Uses Sampled Actions

- Sarsa updates the value of $Q(S_t, A_t)$ based on the **estimated value** of the next action that will **actually be taken** in the next state:

$$Q(S_t, A_t) \leftarrow Q(S_t, A_t) + \alpha \left[R_{t+1} + \gamma \boxed{Q(S_{t+1}, \mathbf{A_{t+1}})} - Q(S_t, A_t)\right]$$

estimate of $v_\pi(S_{t+1}) = \mathbb{E}_\pi \left[Q(S_{t+1}, A_{t+1})\right]$

- *BUT:*
  - We know the **distribution** of A_{t+1} (**what is it?**)
  - The **estimated value** of that action **doesn't depend** on what happens after it is taken (**why?**)
  - Why not estimate $\mathbb{E}_\pi \left[Q(S_{t+1}, A_{t+1})\right]$ by taking **expectation** over A_{t+1}?

Expected Sarsa

Sarsa uses a single sample from $\pi(\cdot \mid S_t)$ to estimate $v_\pi(S_{t+1})$:

$$Q(S_t, A_t) \leftarrow Q(S_t, A_t) + \alpha \left[R_{t+1} + \gamma \boxed{Q(S_{t+1}, \mathbf{A_{t+1}})} - Q(S_t, A_t)\right]$$

Expected Sarsa takes expectation over every possible action:

$$Q(S_t, A_t) \leftarrow Q(S_t, A_t) + \alpha \left[R_{t+1} + \gamma \boxed{\mathbb{E}_{\mathbf{a} \sim \pi(\cdot|S_{t+1})} \left[Q(S_{t+1}, \mathbf{a})\right]} - Q(S_t, A_t)\right]$$

$$= Q(S_t, A_t) + \alpha \left[R_{t+1} + \gamma \sum_{\mathbf{a} \in \mathcal{A}(S_{t+1})} \left[\pi(a \mid S_{t+1}) Q(S_{t+1}, \mathbf{a})\right] - Q(S_t, A_t)\right]$$

Expected Sarsa

Expected Sarsa (on-policy TD control) for estimating $\pi \approx \pi_*$

Algorithm parameters: step size $\alpha \in (0, 1]$, small $\varepsilon > 0$
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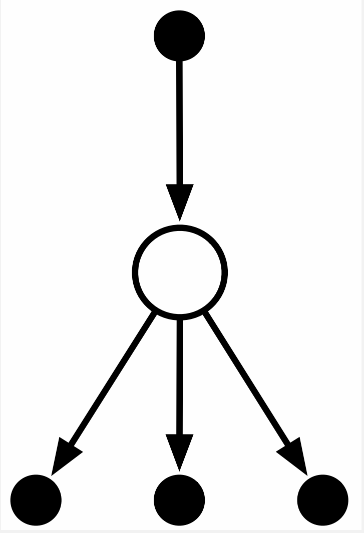
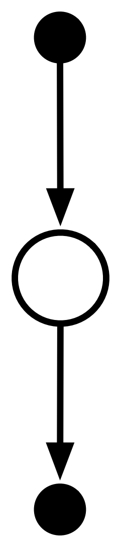
        Take action A, observe R, S'
        $Q(S, A) \leftarrow Q(S, A) + \alpha\left[R + \gamma\left(\sum_a \pi(a \mid S')Q(S', a)\right) - Q(S, A)\right]$
        $S \leftarrow S'$
    until S is terminal

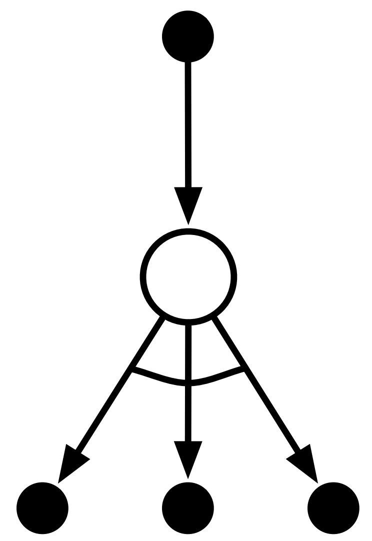


Information Usage

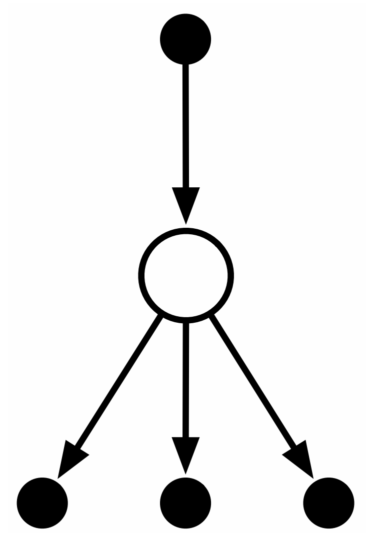
- **Sarsa** uses the actual reward R_t of the actual action A_t taken from an actual state S_t, and the estimated value of the **actual action** A_{t+1} to be taken in the actual next state S_{t+1}



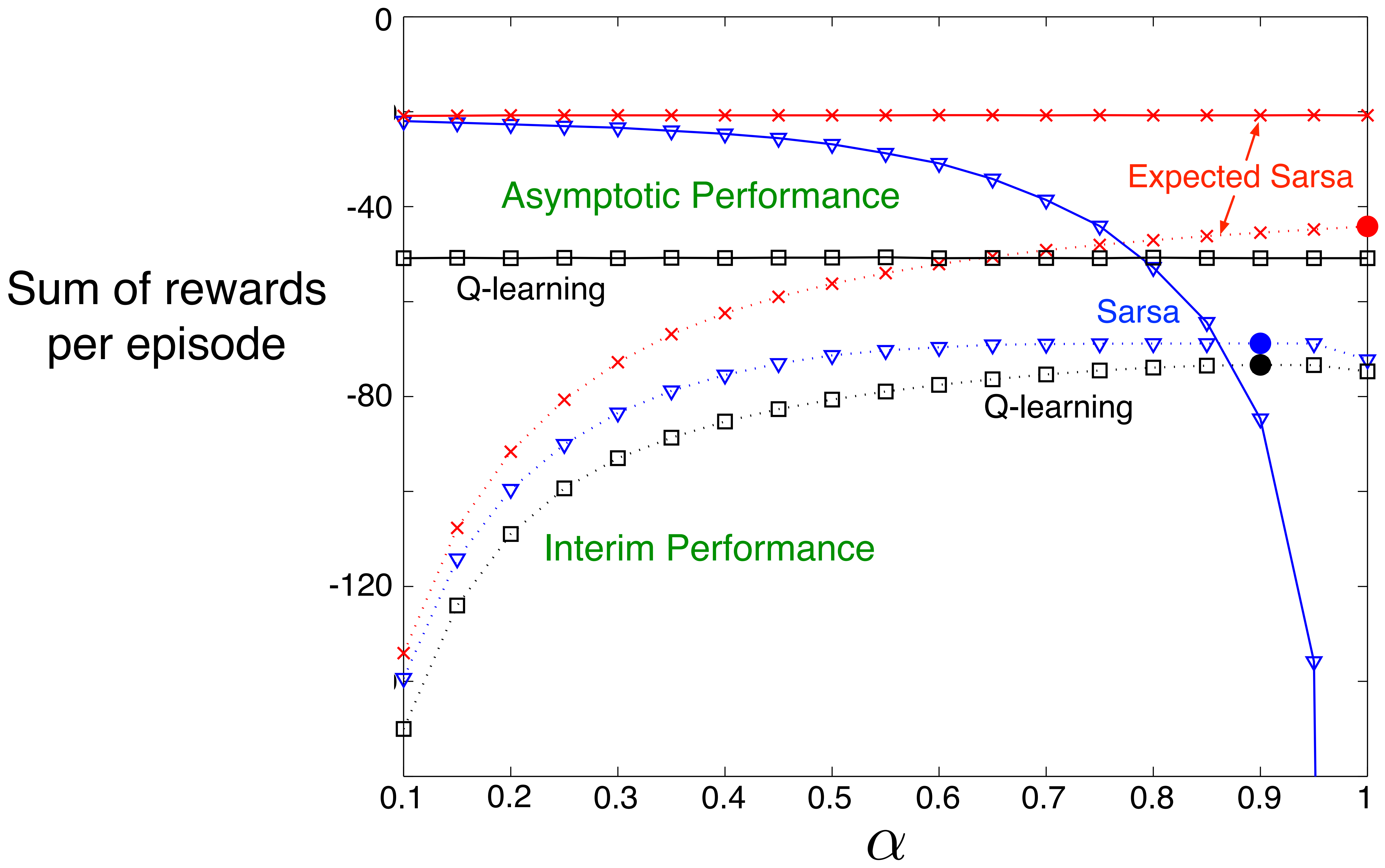
- **Q-Learning** uses the actual reward R_t of the actual action A_t taken from an actual state S_t, and the value of the **highest-estimated-value action** in the actual next state S_{t+1}



- **Expected Sarsa** uses the actual reward R_t of the actual action A_t taken from an actual state S_t, and the **expected estimated value** of next action A_{t+1} to be taken in the actual next state S_{t+1}



Performance on The Cliff, revisited



- For small enough α, Sarsa and Expected Sarsa have same **asymptotic** performance
- For larger α, Expected Sarsa has increasingly high **interim** performance, whereas Sarsa has increasingly poor interim performance (**why?**)

Summary

- Temporal Difference Learning **bootstraps** *and* learns from **experience**
  - Dynamic programming bootstraps, but doesn't learn from experience (requires full dynamics)
  - Monte Carlo learns from experience, but doesn't bootstrap
- Prediction: **TD(0) algorithm**
- **Sarsa** estimates action-values of **actual ϵ-greedy policy**
  - **Expected Sarsa** estimates action-values of ϵ-greedy policy
- **Q-Learning** estimates action-values of **optimal** policy while **executing** an **ϵ-greedy** policy