1. (**Short Answers**; 10 points)

   (a) **[2 points]** State Bayes' rule.

   $$\Pr(h|e) = \frac{\Pr(e|h)\Pr(h)}{\Pr(e)}$$

   (of course the names of the variables need not be h, e)

   (b) **[2 points]** Define conditional independence.
   X and Y are independent conditional on Z if $\Pr(X \mid Y, Z) = \Pr(X \mid Z)$ for all values of X, Y, and Z.

   (c) **[2 points]** What guarantee does a belief network make about conditional independence?
   Every node is independent of its non-descendants conditional on only its parents.

   (d) **[2 points]** Explain why variable elimination can be more efficient than just computing the full joint distribution and then normalizing. Because by eliminating variables one at a time, variable elimination can often avoid repeated operations. (Also acceptable to say that it can operate on smaller factor tables, or not have to process as many rows even though it processes more tables.)

   (e) **[2 points]** Let X be a random variable with the following distribution:

   $$\Pr(X = 2) = 1/2 \quad \Pr(X = 4) = 1/4 \quad \Pr(X = 8) = 1/4$$

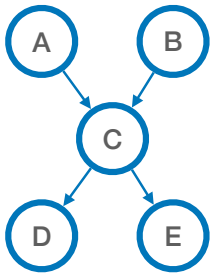
   What is the expected value $\mathbb{E}[X]$?

   $$\begin{aligned}\mathbb{E}[X] &= \sum_{x \in \mathrm{dom}(X)} \Pr(X = x)x \\ &= \frac{1}{2}1 + \frac{1}{4}4 + \frac{1}{4}8 \\ &= 1 + 1 + 2 = \mathbf{4}\end{aligned}$$

2. (**Uncertainty**; 14 points)

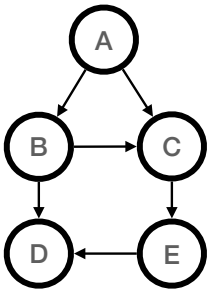
   (a) **[3 points]** Draw a belief network that represents the following factoring, *or* list each of the nodes of the network along with its parents:

   $$P(A, B, C, D, E) = P(A)P(B)P(C \mid A, B)P(D \mid C)P(E \mid C)$$



(b) **[3 points]** What factorization of the joint distribution $P(A, B, C, D, E)$ does the belief network below represent?

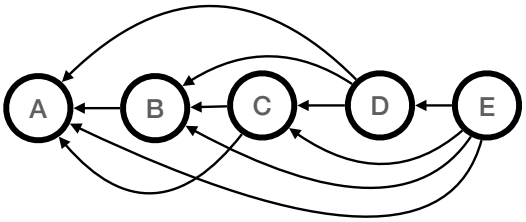
In this network, A has no parents, B's parent is A, C's parents are A and B, D's parents are B and E, and E's parent is C.



$P(A)P(B \mid A)P(C \mid A, B)P(D \mid B, E)P(E \mid C)$

(c) **[3 points]** Is the belief network below also consistent with the same joint distribution? Why or why not? (Hint: What factoring does the belief network below represent?)

In this network, E has no parents, D's parent is E, C's parents are D and E, B's parents are C, D, and E, and A's parents are B, C, D, and E.



Yes. This belief network represents the factoring

$P(A \mid B, C, D, E)P(B \mid C, D, E)P(C \mid D, E)P(D \mid E)P(E),$

which is a valid factoring for *any* joint distribution $P(A, B, C, D, E)$ by the chain rule.

(d) **[2 points]** List the factors that would be constructed as the first step of variable elimination for the belief network from question (2a). $\{f_1(A), f_2(B), f_3(C, A, B), f_4(D, C), f_5(E, C)\}$ (note: obviously the names and order of arguments needn't match)

(e) **[3 points]** List the new factor, and the operations used to create it, when the variable elimination algorithm eliminates A from the list of factors in question (2d). $f_6(B, C) = \sum_A (f_1 \times f_3)$

3. (**Variable Elimination**; 9 points)

Consider the following factors:

B	C	f_1
0	0	.8
0	1	.3
1	0	.1
1	1	.2

A	C	f_2
0	0	.25
0	1	.1
1	0	.5
1	1	.2

(a) **[3 points]** What is the result of the operation $f_3 = \sum_B f_1$?

C	f_3
0	.9
1	.5

(b) **[3 points]** What is the result of the operation $f_4 = (f_1)_{C=0}$?

B	f_4
0	.8
1	.1

(c) **[3 points]** What is the result of the operation $f_5 = f_1 \times f_2$?

A	B	C	f_5
0	0	0	.2
0	0	1	.03
0	1	0	.025
0	1	1	.02
1	0	0	.4
1	0	1	.06
1	1	0	.05
1	1	1	.04