Monte Carlo Prediction & Control

CMPUT 261: Introduction to Artificial Intelligence

S&B §5.0-5.5, 5.7

Lecture Outline

1. Recap & Logistics
2. Monte Carlo Prediction
3. Estimating Action Values
4. Monte Carlo Control
5. Importance Sampling
6. Off-Policy Monte Carlo Control

After this lecture, you should be able to:

- explain how Monte Carlo estimation for state values works
- trace an execution of first-visit Monte Carlo Prediction
- explain the difference between prediction and control
- define on-policy vs. off-policy learning
- define a behaviour policy
- define exploring starts
- explain what problem exploring starts solve
- define an epsilon-soft policy
- explain what problem epsilon-soft policies solve

Previous Lecture Summary

- An **optimal policy** has higher state value than any other policy **at every state**
- A policy's state-value function can be computed by **iterating** an **expected update** based on the Bellman equation
- Given any policy π, we can compute a **greedy improvement** π' by choosing highest expected value action based on v_π
- **Policy iteration:** Repeat:
  Greedy improvement using v_π, then recompute v_π
- **Value iteration:** Repeat:
  Recompute v_π **by assuming** greedy improvement at every update

Recap: In-Place Iterative Policy Evaluation

Iterative Policy Evaluation, for estimating $V \approx v_\pi$

Input π, the policy to be evaluated
Algorithm parameter: a small threshold $\theta > 0$ determining accuracy of estimation
Initialize $V(s)$, for all $s \in \mathcal{S}^+$, arbitrarily except that $V(terminal) = 0$

Loop:
  $\Delta \leftarrow 0$
  Loop for each $s \in \mathcal{S}$:
    $v \leftarrow V(s)$
    $V(s) \leftarrow \sum_a \pi(a|s) \sum_{s',r} p(s', r \,|\, s, a)\big[r + \gamma V(s')\big]$
    $\Delta \leftarrow \max(\Delta, |v - V(s)|)$
until $\Delta < \theta$

- These are **expected updates:** Based on a weighted average (expectation) of **all possible next states**

Recap: Policy Improvement Theorem

Theorem:
Let π and π' be any pair of deterministic policies.

If $q_\pi(s, \pi'(s)) \geq v_\pi(s) \quad \forall s \in \mathcal{S}$,

then $v_{\pi'}(s) \geq v_\pi(s) \quad \forall s \in \mathcal{S}$.

If you are never worse off **at any state** by following π' for **one step** and then following π forever after, then following π' **forever** has a higher expected value **at every state**.

Recap: Policy Iteration

$$\pi_0 \xrightarrow{\mathrm{E}} v_{\pi_0} \xrightarrow{\mathrm{I}} \pi_1 \xrightarrow{\mathrm{E}} v_{\pi_1} \xrightarrow{\mathrm{I}} \pi_2 \xrightarrow{\mathrm{E}} \cdots \xrightarrow{\mathrm{I}} \pi_* \xrightarrow{\mathrm{E}} v_*$$

An arrow from π_0 to v_{π_0} is labelled "E", an arrow from v_{π_0} to π_1 is labelled "I", an arrow from π_1 to v_{π_1} is labelled "E", an arrow from v_{π_1} to π_2 is labelled "I", an arrow from π_2 to ... is labelled "E", an arrow from ... to π_* is labelled "I", an arrow from π_* to v_{π_*} is labelled "E".

Policy Iteration (using iterative policy evaluation) for estimating $\pi \approx \pi_*$

1. Initialization
   $V(s) \in \mathbb{R}$ and $\pi(s) \in \mathcal{A}(s)$ arbitrarily for all $s \in \mathcal{S}$

2. Policy Evaluation
   Loop:
   - $\Delta \leftarrow 0$
   - Loop for each $s \in \mathcal{S}$:
     - $v \leftarrow V(s)$
     - $V(s) \leftarrow \sum_{s',r} p(s', r \,|\, s, \pi(s)) \big[r + \gamma V(s') \big]$
     - $\Delta \leftarrow \max(\Delta, |v - V(s)|)$

   until $\Delta < \theta$ (a small positive number determining the accuracy of estimation)

3. Policy Improvement
   policy-stable $\leftarrow$ *true*
   For each $s \in \mathcal{S}$:
   - *old-action* $\leftarrow \pi(s)$
   - $\pi(s) \leftarrow \operatorname{arg\,max}_a \sum_{s',r} p(s', r \,|\, s, a) \big[r + \gamma V(s') \big]$
   - If *old-action* $\neq \pi(s)$, then *policy-stable* $\leftarrow$ *false*

   If *policy-stable*, then stop and return $V \approx v_*$ and $\pi \approx \pi_*$; else go to 2

Value Iteration

Value iteration **interleaves** the estimation and improvement steps:

$$v_{k+1}(s) \doteq \max_a \mathbb{E}\left[R_{t+1} + \gamma v_k(S_{t+1}) \,|\, S_t = s, A_t = a\right]$$
$$= \max_a \sum_{s',r} p(s', r \,|\, s, a)\left[r + \gamma v_k(s')\right]$$

Value Iteration, for estimating $\pi \approx \pi_*$

Algorithm parameter: a small threshold $\theta > 0$ determining accuracy of estimation
Initialize $V(s)$, for all $s \in \mathcal{S}^+$, arbitrarily except that $V(terminal) = 0$

Loop:
| $\Delta \leftarrow 0$
| Loop for each $s \in \mathcal{S}$:
| $v \leftarrow V(s)$
| $V(s) \leftarrow \max_a \sum_{s',r} p(s', r \,|\, s, a)\left[r + \gamma V(s')\right]$
| $\Delta \leftarrow \max(\Delta, |v - V(s)|)$
until $\Delta < \theta$

Output a deterministic policy, $\pi \approx \pi_*$, such that
$\pi(s) = \operatorname{arg\,max}_a \sum_{s',r} p(s', r \,|\, s, a)\left[r + \gamma V(s')\right]$

Example: Blackjack

- Player gets two cards, dealer gets 1
- Player can `hit` (get a new card) as many times as they like, or `stick` (stop hitting)
- After the player is done, the dealer hits / sticks according to a fixed rule
- Whoever has the most points (sum of card values) wins
- But, if you have more than 21 points, you **lose immediately** ("bust")

Simulating Blackjack

- Given a policy for the player, it is **very easy** to simulate a game of Blackjack
    - (Treat both the cards and the dealer as part of the environment)
- **Question:** Is it easy to **compute** the full **dynamics**?
- **Question:** Is it easy to run **iterative policy evaluation**?

Experience vs. Expectation

- In order to compute **expected updates**, we need to know the exact **probability** of **every** possible transition

- Often we don't have access to the full probability distribution, but we do have access to **samples of experience**

1. **Actual experience:** We want to learn based on interactions with a **real environment**, without knowing its dynamics

2. **Simulated experience:** We can **simulate** the dynamics, but we don't have an **explicit representation** of transition probabilities, or there are **too many states**

Monte Carlo Estimation

- Instead of estimating expectations by a **weighted sum** over **all possibilities**, estimate expectation by **averaging** over a **sample** drawn from the distribution:

$$\mathbb{E}[X] = \sum_x f(x)x \approx \frac{1}{n}\sum_{i=1}^{n} x_i \quad \text{where } x_i \sim f$$

Monte Carlo Prediction

- Use a **large sample** of **episodes** generated by a policy π to estimate the state-values $v_\pi(s)$ for each state s
  - We will consider only **episodic** tasks for now
- **Question:** What is the **return** G_t for state $S_t = s$ in a given episode?
- We can estimate the expected return $v_\pi(s) = \mathbb{E}[G_t \mid S_t = s]$ by averaging the returns for that state in every episode containing a visit to s

First-visit Monte Carlo Prediction

First-visit MC prediction, for estimating $V \approx v_\pi$

Input: a policy π to be evaluated

Initialize:

  $V(s) \in \mathbb{R}$, arbitrarily, for all $s \in \mathcal{S}$

  $Returns(s) \leftarrow$ an empty list, for all $s \in \mathcal{S}$

Loop forever (for each episode):

  Generate an episode following π: $S_0, A_0, R_1, S_1, A_1, R_2, \ldots, S_{T-1}, A_{T-1}, R_T$

  $G \leftarrow 0$

  Loop for each step of episode, $t = T-1, T-2, \ldots, 0$:

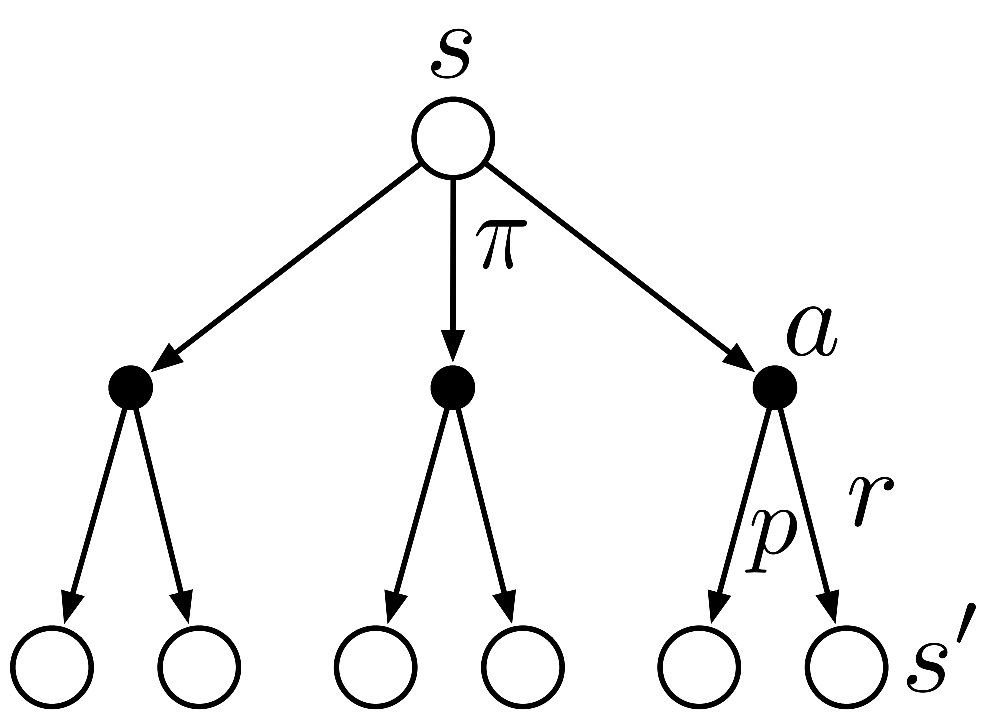
    $G \leftarrow \gamma G + R_{t+1}$

    Unless S_t appears in $S_0, S_1, \ldots, S_{t-1}$:

      Append G to $Returns(S_t)$

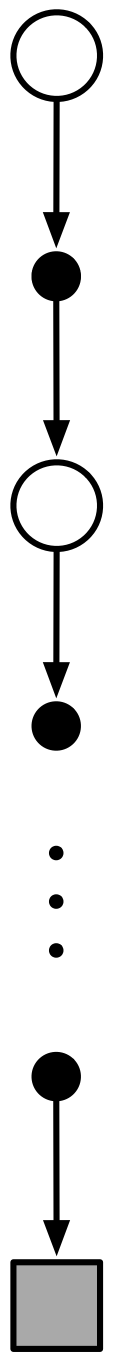
      $V(S_t) \leftarrow \text{average}(Returns(S_t))$

Monte Carlo vs. Dynamic Programming



An open circle "s" has outgoing edges leading to three filled circles "a", each of which has two outgoing edges labelled "p" and "r" to open circles labelled "s'".

- **Iterative policy evaluation** uses the estimates of the **next state's** value to update the value of this state
  - Only needs to compute a **single transition** to update a state's estimate
- **Monte Carlo** estimate of each state's value is **independent** from estimates of **other states'** values
  - Needs the **entire episode** to compute an update
  - Can focus on evaluating a **subset of states** if desired



An open circle has a single outgoing edge to a filled circle which has a single outgoing edge to an open circle which has a single outgoing edge to a filled circle, ellipsis, to a filled circle that has a single outgoing edge to a filled square (i.e., terminal state).

Control vs. Prediction

- **Prediction:** estimate the value of states and/or actions given some fixed policy π
- **Control:** estimate an optimal policy

Estimating Action Values

- When we know the **dynamics** $p(s', r \mid s, a)$, an estimate of **state values** is sufficient to determine a good **policy**:
  - Choose the action that gives the best combination of **reward** and **next-state value**:

$$\hat{a}^* = \arg\max_{a \in \mathcal{A}} \sum_{s',r} p(s', r \mid s, a)[r + \gamma \hat{v}(s')]$$

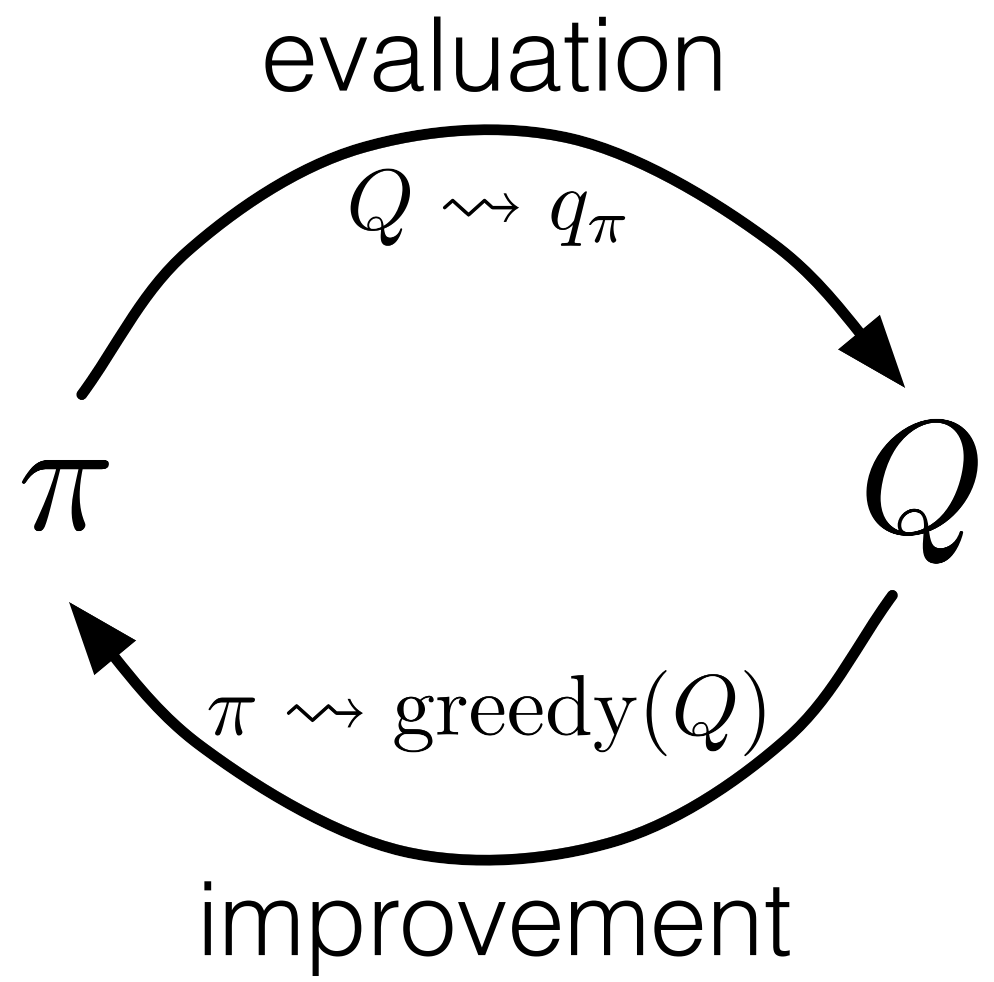
  - This is why Value Iteration only explicitly estimates state values, not policy
- If we *don't* know the dynamics, state values are **not enough**
  - To estimate a good policy, we need an **explicit** estimate of **action values**

Exploring Starts for Action-Value Estimation

- We can just run first-visit Monte Carlo and approximate the returns to each **state-action pair**
- **Question:** What do we do about state-action pairs that are **never visited**?
  - If the current policy π never selects an action a from a state s, then Monte Carlo can't estimate its value
- **Exploring starts assumption:**
  - Every episode **starts** at a random state-action pair S_0, A_0
  - **Every pair** has a positive probability of being selected for a start

Monte Carlo Control

Monte Carlo control can be used for **policy iteration**:



An edge from π to Q is labelled "evaluation: $Q \rightsquigarrow q_\pi$", and an edge back from Q to π is labelled "improvement:$\pi \rightsquigarrow \text{greedy}(Q)$"

$$\pi_0 \xrightarrow{\text{E}} q_{\pi_0} \xrightarrow{\text{I}} \pi_1 \xrightarrow{\text{E}} q_{\pi_1} \xrightarrow{\text{I}} \pi_2 \xrightarrow{\text{E}} \cdots \xrightarrow{\text{I}} \pi_* \xrightarrow{\text{E}} q_*$$

An arrow from π_0 to v_{π_0} is labelled "E", an arrow from v_{π_0} to π_1 is labelled "I", an arrow from π_1 to $v_{\pi_1}$is labelled "E", an arrow from v_{π_1} to π_2 is labelled "I", an arrow from π_2 to ... is labelled "E", an arrow from ... to π_* is labelled "I", an arrow from π_* to v_{π_*} is labelled "E".

Monte Carlo Control with Exploring Starts

Monte Carlo ES (Exploring Starts), for estimating $\pi \approx \pi_*$

Initialize:
  $\pi(s) \in \mathcal{A}(s)$ (arbitrarily), for all $s \in \mathcal{S}$
  $Q(s,a) \in \mathbb{R}$ (arbitrarily), for all $s \in \mathcal{S}$, $a \in \mathcal{A}(s)$
  $Returns(s,a) \leftarrow$ empty list, for all $s \in \mathcal{S}$, $a \in \mathcal{A}(s)$

Loop forever (for each episode):
  Choose $S_0 \in \mathcal{S}$, $A_0 \in \mathcal{A}(S_0)$ randomly such that all pairs have probability > 0
  Generate an episode from S_0, A_0, following π: $S_0, A_0, R_1, \ldots, S_{T-1}, A_{T-1}, R_T$
  $G \leftarrow 0$
  Loop for each step of episode, $t = T-1, T-2, \ldots, 0$:
    $G \leftarrow \gamma G + R_{t+1}$
    Unless the pair S_t, A_t appears in $S_0, A_0, S_1, A_1 \ldots, S_{t-1}, A_{t-1}$:
      Append G to $Returns(S_t, A_t)$
      $Q(S_t, A_t) \leftarrow \text{average}(Returns(S_t, A_t))$
      $\pi(S_t) \leftarrow \operatorname{argmax}_a Q(S_t, a)$

Question: What unlikely assumptions does this rely upon?

ε-Soft Policies

- The **exploring starts** assumption requires that we see **every** state-action pair with positive probability
    - Even if π **never** chooses a from state s
- Another approach: Simply **force** π to (sometimes) choose a!
- An **ϵ-soft policy** is one for which $\pi(a \mid s) \geq \dfrac{\epsilon}{|\mathcal{A}(s)|} \quad \forall s, a$
- **Example:** **ϵ-greedy policy**

$$\pi(a \mid s) = \begin{cases} \frac{\epsilon}{|\mathcal{A}(s)|} & \text{if } a \notin \arg\max_a Q(s,a), \\ (1-\epsilon) + \frac{\epsilon}{|\mathcal{A}(s)|} & \text{otherwise.} \end{cases}$$

Monte Carlo Control w/out Exploring Starts

On-policy first-visit MC control (for ε-soft policies), estimates $\pi \approx \pi_*$

Algorithm parameter: small $\varepsilon > 0$

Initialize:

$\pi \leftarrow$ an arbitrary ε-soft policy
$Q(s,a) \in \mathbb{R}$ (arbitrarily), for all $s \in \mathcal{S}$, $a \in \mathcal{A}(s)$
$Returns(s,a) \leftarrow$ empty list, for all $s \in \mathcal{S}$, $a \in \mathcal{A}(s)$

Repeat forever (for each episode):

Generate an episode following π: $S_0, A_0, R_1, \ldots, S_{T-1}, A_{T-1}, R_T$
$G \leftarrow 0$
Loop for each step of episode, $t = T-1, T-2, \ldots, 0$:
$G \leftarrow \gamma G + R_{t+1}$
Unless the pair S_t, A_t appears in $S_0, A_0, S_1, A_1 \ldots, S_{t-1}, A_{t-1}$:
Append G to $Returns(S_t, A_t)$
$Q(S_t, A_t) \leftarrow \text{average}(Returns(S_t, A_t))$
$A^* \leftarrow \arg\max_a Q(S_t, a)$ (with ties broken arbitrarily)
For all $a \in \mathcal{A}(S_t)$:

$$\pi(a|S_t) \leftarrow \begin{cases} 1 - \varepsilon + \varepsilon/|\mathcal{A}(S_t)| & \text{if } a = A^* \\ \varepsilon/|\mathcal{A}(S_t)| & \text{if } a \neq A^* \end{cases}$$

Monte Carlo Control w/out Exploring Starts

On-policy first-visit MC control (for ε-soft policies), estimates $\pi \approx \pi_*$

Algorithm parameter: small $\varepsilon > 0$

Initialize:

$\pi \leftarrow$ an arbitrary ε-soft policy

$Q(s,a) \in \mathbb{R}$ (arbitrarily), for all $s \in \mathcal{S}$, $a \in \mathcal{A}(s)$

$Returns(s,a) \leftarrow$ empty list, for all $s \in \mathcal{S}$, $a \in \mathcal{A}(s)$

Repeat forever (for each episode):

Generate an episode following π: $S_0, A_0, R_1, \ldots, S_{T-1}, A_{T-1}, R_T$

$G \leftarrow 0$

Loop for each step of episode, $t = T-1, T-2, \ldots, 0$:

$G \leftarrow \gamma G + R_{t+1}$

Unless the pair S_t, A_t appears in $S_0, A_0, S_1, A_1 \ldots, S_{t-1}, A_{t-1}$:

Append G to $Returns(S_t, A_t)$

$Q(S_t, A_t) \leftarrow \text{average}(Returns(S_t, A_t))$

$A^* \leftarrow \arg\max_a Q(S_t, a)$ (with ties broken arbitrarily)

For all $a \in \mathcal{A}(S_t)$:

$$\pi(a|S_t) \leftarrow \begin{cases} 1 - \varepsilon + \varepsilon/|\mathcal{A}(S_t)| & \text{if } a = A^* \\ \varepsilon/|\mathcal{A}(S_t)| & \text{if } a \neq A^* \end{cases}$$

Question:

Will this procedure converge to the optimal policy π^*?

Why or why not?

Importance Sampling

- **Monte Carlo sampling:** use samples from the **target** distribution to estimate expectations
- **Importance sampling:** Use samples from **proposal** distribution to estimate expectations of **target** distribution by **reweighting** samples

$$\mathbb{E}[X] = \sum_x f(x)x = \sum_x \frac{g(x)}{g(x)} f(x)x = \sum_x g(x)\frac{f(x)}{g(x)}x \approx \frac{1}{n}\sum_{x_i \sim g} \boxed{\frac{f(x_i)}{g(x_i)}} x_i$$

Importance sampling ratio

Off-Policy Prediction via Importance Sampling

Definition:
Off-policy learning means using data generated by a **behaviour policy** to learn about a distinct **target policy**.

Proposal distribution

Target distribution

Off-Policy Monte Carlo Prediction

- Generate episodes using **behaviour policy** b
- Take **weighted** average of returns to state s over all the episodes containing a visit to s to estimate $v_\pi(s)$
  - Weighed by **importance sampling ratio** of trajectory starting from $S_t = s$ until the end of the episode:

$$\rho_{t:T-1} \doteq \frac{\Pr[A_t, S_{t+1}, \ldots, S_T \mid S_t, A_{t:T-1} \sim \pi]}{\Pr[A_t, S_{t+1}, \ldots, S_T \mid S_t, A_{t:T-1} \sim b]}$$

Importance Sampling Ratios for Trajectories

- **Probability of a trajectory** $A_t, S_{t+1}, A_{t+1}, \ldots, S_T$ from S_t:

$$\Pr[A_t, S_{t+1}, \ldots, S_T \mid S_t, A_{t:T-1} \sim \pi] =$$

$$\pi(A_t \mid S_t) p(S_{t+1} \mid S_t, A_t) \pi(A_{t+1} \mid S_{t+1}) \ldots p(S_T \mid S_{T-1}, A_{T-1})$$

- **Importance sampling ratio** for a trajectory $A_t, S_{t+1}, A_{t+1}, \ldots, S_T$ from S_t:

$$\rho_{t:T-1} \doteq \frac{\prod_{k=t}^{T-1} \pi(A_k \mid S_k) p(S_{k+1} \mid S_k, A_k)}{\prod_{k=t}^{T-1} b(A_k \mid S_k) p(S_{k+1} \mid S_k, A_k)} = \frac{\prod_{k=t}^{T-1} \pi(A_k \mid S_k)}{\prod_{k=t}^{T-1} b(A_k \mid S_k)}$$

$= 1$

Ordinary vs. Weighted Importance Sampling

- **Ordinary importance sampling:**

$$V(s) \doteq \frac{1}{n} \sum_{i=1}^{n} \rho_{t(s,i):T(i)-1} G_{i,t}$$

- **Weighted importance sampling:**

$$V(s) \doteq \frac{\sum_{i=1}^{n} \rho_{t(s,i):T(i)-1} G_{i,t}}{\sum_{i=1}^{n} \rho_{t(s,i):T(i)-1}}$$

Example: Ordinary vs. Weighted Importance Sampling for Blackjack

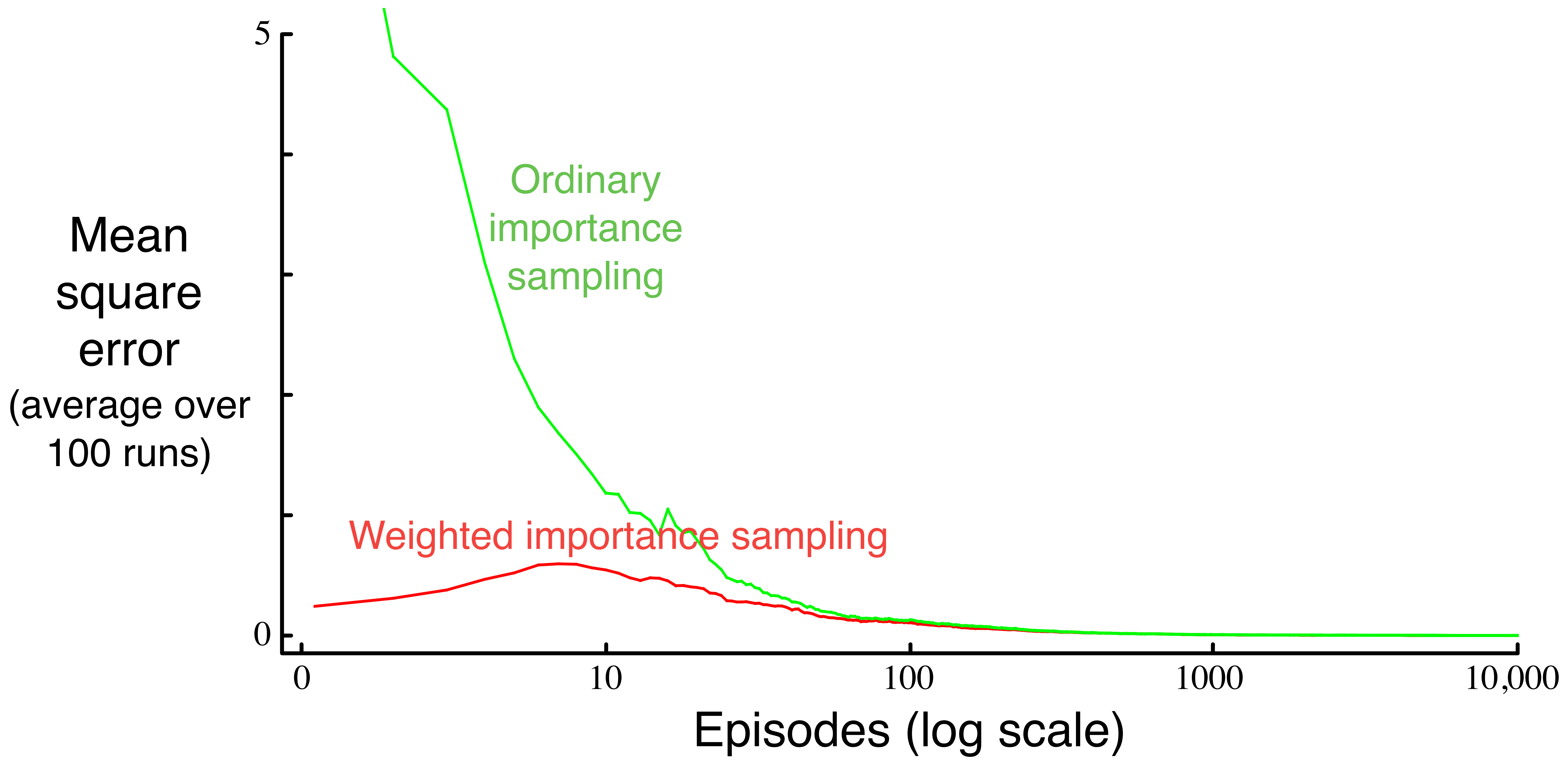


Figure 5.3: Weighted importance sampling produces lower error estimates of the value of a single blackjack state from off-policy episodes. ■

Figure with mean square error (averaged over 100 runs) on y-axis and episodes (log scale) on x-axis. Ordinary importance sampling starts high but converges to a low error; weighted importance sampling starts relatively low, gets slightly higher, and then overlaps the ordinary importance sampling trace after 100 episodes.

(Image: Sutton & Barto, 2018)

Off-Policy Monte Carlo **Prediction**

Off-policy MC prediction (policy evaluation) for estimating $Q \approx q_\pi$

Input: an arbitrary target policy π
Initialize, for all $s \in \mathcal{S}$, $a \in \mathcal{A}(s)$:
    $Q(s, a) \in \mathbb{R}$ (arbitrarily)
    $C(s, a) \leftarrow 0$

Loop forever (for each episode):
    $b \leftarrow$ any policy with coverage of π
    Generate an episode following b: $S_0, A_0, R_1, \ldots, S_{T-1}, A_{T-1}, R_T$
    $G \leftarrow 0$
    $W \leftarrow 1$
    Loop for each step of episode, $t = T-1, T-2, \ldots, 0$, while $W \neq 0$:
        $G \leftarrow \gamma G + R_{t+1}$
        $C(S_t, A_t) \leftarrow C(S_t, A_t) + W$
        $Q(S_t, A_t) \leftarrow Q(S_t, A_t) + \frac{W}{C(S_t, A_t)} [G - Q(S_t, A_t)]$
        $W \leftarrow W \frac{\pi(A_t|S_t)}{b(A_t|S_t)}$

Off-Policy Monte Carlo Control

Off-policy MC control, for estimating $\pi \approx \pi_*$

Initialize, for all $s \in \mathcal{S}$, $a \in \mathcal{A}(s)$:
- $Q(s,a) \in \mathbb{R}$ (arbitrarily)
- $C(s,a) \leftarrow 0$
- $\pi(s) \leftarrow \arg\max_a Q(s,a)$ (with ties broken consistently)

Loop forever (for each episode):
- $b \leftarrow$ any soft policy
- Generate an episode using b: $S_0, A_0, R_1, \ldots, S_{T-1}, A_{T-1}, R_T$
- $G \leftarrow 0$
- $W \leftarrow 1$
- Loop for each step of episode, $t = T-1, T-2, \ldots, 0$:
  - $G \leftarrow \gamma G + R_{t+1}$
  - $C(S_t, A_t) \leftarrow C(S_t, A_t) + W$
  - $Q(S_t, A_t) \leftarrow Q(S_t, A_t) + \frac{W}{C(S_t, A_t)} [G - Q(S_t, A_t)]$
  - $\pi(S_t) \leftarrow \arg\max_a Q(S_t, a)$ (with ties broken consistently)
  - If $A_t \neq \pi(S_t)$ then exit inner Loop (proceed to next episode)
  - $W \leftarrow W \frac{1}{b(A_t|S_t)}$

Off-Policy Monte Carlo Control

Off-policy MC control, for estimating $\pi \approx \pi_*$

Initialize, for all $s \in \mathcal{S}$, $a \in \mathcal{A}(s)$:
  $Q(s,a) \in \mathbb{R}$ (arbitrarily)
  $C(s,a) \leftarrow 0$
  $\pi(s) \leftarrow \arg\max_a Q(s,a)$ (wi

Loop forever (for each episode):
  $b \leftarrow$ any soft policy
  Generate an episode using b:
  $G \leftarrow 0$
  $W \leftarrow 1$
  Loop for each step of episode, $t = T-1, T-2, \ldots, 0$:
    $G \leftarrow \gamma G + R_{t+1}$
    $C(S_t, A_t) \leftarrow C(S_t, A_t) + W$
    $Q(S_t, A_t) \leftarrow \boxed{Q(S_t, A_t) + \frac{W}{C(S_t,A_t)}\left[G - Q(S_t, A_t)\right]}$
    $\pi(S_t) \leftarrow \arg\max_a Q(S_t, a)$ (with ties broken consistently)
    If $A_t \neq \pi(S_t)$ then exit inner Loop (proceed to next episode)
    $W \leftarrow W \frac{1}{b(A_t|S_t)}$

$$Q_n = \frac{\sum_{i=1}^{n} W_i G_i}{\sum_{i=1}^{n} W_i} = \frac{\sum_{i=1}^{n} W_i G_i}{C - W}$$

$$Q_{n+1} = \frac{\sum_{i=1}^{n+1} W_i G_i}{\sum_{i=1}^{n+1} W_i} = \frac{(C-W)Q_n + WG}{C}$$

$$= \frac{C}{C}Q_n - \frac{W}{C}Q_n + \frac{W}{C}G = Q_n + \frac{W}{C}\left[G - Q_n\right]$$

Questions:

1. Will this procedure converge to the optimal policy π^*?
2. Why do we break when $A_t \neq \pi(S_t)$?
3. Why do the weights W not involve $\pi(A_t \mid S_t)$?

Summary

- **Monte Carlo estimation** estimates values by averaging returns over **sample episodes**
  - Does not require access to full model of **dynamics**
  - Does require access to an entire **episode** for each sample
- Estimating **action values** requires either **exploring starts** or a **soft policy** (e.g., ϵ-greedy)
- **Off-policy learning** is the estimation of value functions for a **target policy** based on episodes generated by a different **behaviour policy**
  - **Importance sampling** is one way to perform off-policy learning
  - **Weighted** importance sampling has lower **variance** than **ordinary** importance sampling
- **Off-policy control** is learning the **optimal policy** (target policy) using episodes from a **behaviour policy**