Three Proof Techniques for Constructing the Surreal Numbers from Nim

CMPUT 355: Games, Puzzles, and Algorithms

Imperfect Information Games

CMPUT 355: Games, Puzzles, and Algorithms

Lecture Outline

1. Logistics & Recap
2. Hidden actions
3. Imperfect information games
4. Perfect recall

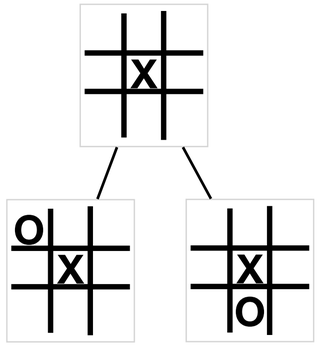
Final Exam Logistics

- **Final exam: Wed April 15**
  - **CCIS L2-190** (this room)
  - **1:00pm** (*not* this time)
  - Format: like a **quiz**, but longer!
  - Cumulative: covers the **whole semester**
  - *Very slight* emphasis on post-quiz-5 (maybe one extra question)
- **Practice material:**
  - No practice final (but: finals from past offerings are available here)
  - Practice questions #1-5 will be very relevant to the final
  - Additional **practice questions #6** to be released Tue **Apr 7** (solutions **Apr 9**)

Recap: Game Types

So far we have looked at three different models of game play:

1. **Perfect information sequential games**
   - Game state perfectly known
   - Other player's move revealed before own move
   - Randomization is **never necessary**



2. **Normal form games**
   - Only a single, simultaneous move
   - Other player's move not revealed until after own move
   - **Randomization** sometimes needed

	Rock	Paper	Scissor
Rock	0,0	-1,1	1,-1
Paper	1,-1	0,0	-1,1
Scissor	-1,1	1,-1	0,0

3. **Repeated normal form games**
   - Other player's **previous** move revealed before own move, but not **current** move

	c	d
C	-1	-4
D	0,	-3

and then

	c	d
C	-1	-4
D	0,	-3

and then

	c	d
C	-1	-4
D	0,	-3

and then

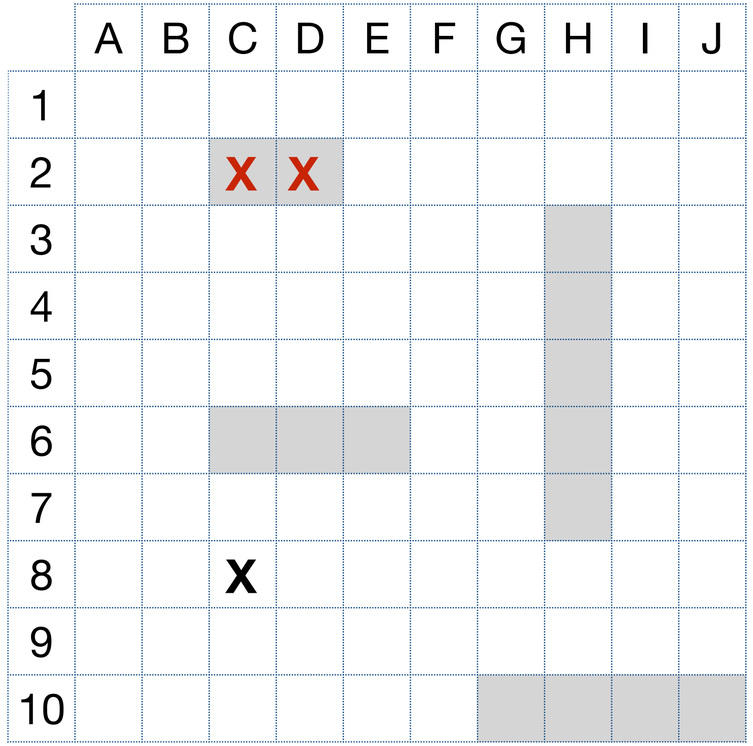
	c	d
C	-1	-4
D	0,	-3

and then

	c	d
C	-1	-4
D	0,	-3

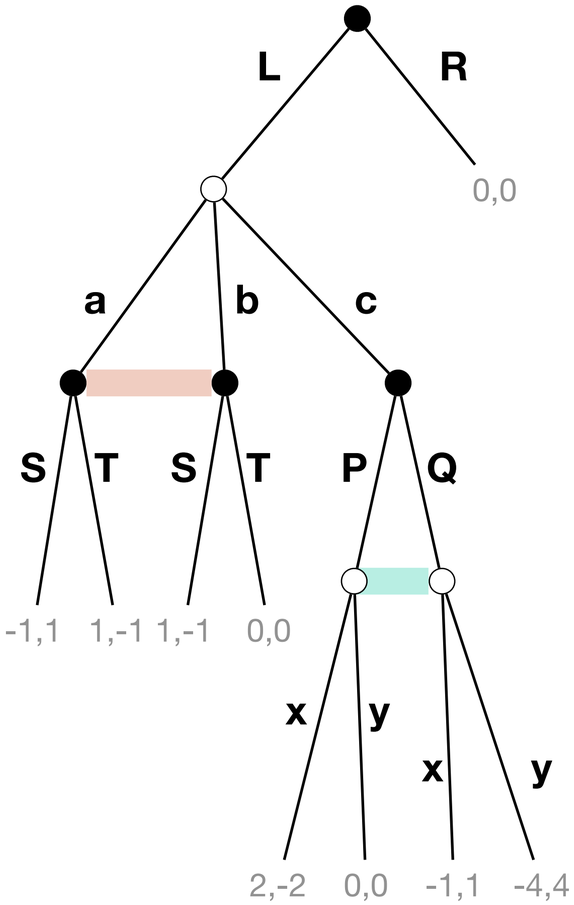
Hidden Actions

- In perfect information sequential games, actions are visible and sequential
- In normal form games, actions are hidden but simultaneous
- But in some games, actions are **both hidden** and **sequential**
- **Example: Battleship**
  - Phase 1: players **secretly** choose their own ship locations
  - Phase 2: players **publicly** shoot at each other's grid
  - After each shot, **partial information** about ship locations is revealed:
    - No ship there, or
    - Part of a ship at that location, or
    - Ship sunk



Information Sets

- An **imperfect information (extensive form) game** is simply a perfect information game where states are grouped together into **information sets**
- When a player arrives at a state inside an information set, they don't know **which state** they are at
- Every state in an information set must have the **same** available actions (**why?**)
- After history **L a**,
  - P1 can't tell if P2 played **a** or **b**, but they know they did **not** play **c**
- After **L c Q**, P2 doesn't know if P1 played **P** or **Q**
- **Question:** What is a **pure strategy** for an imperfect information game?
- **Question:** What are the information sets for each player in this game?

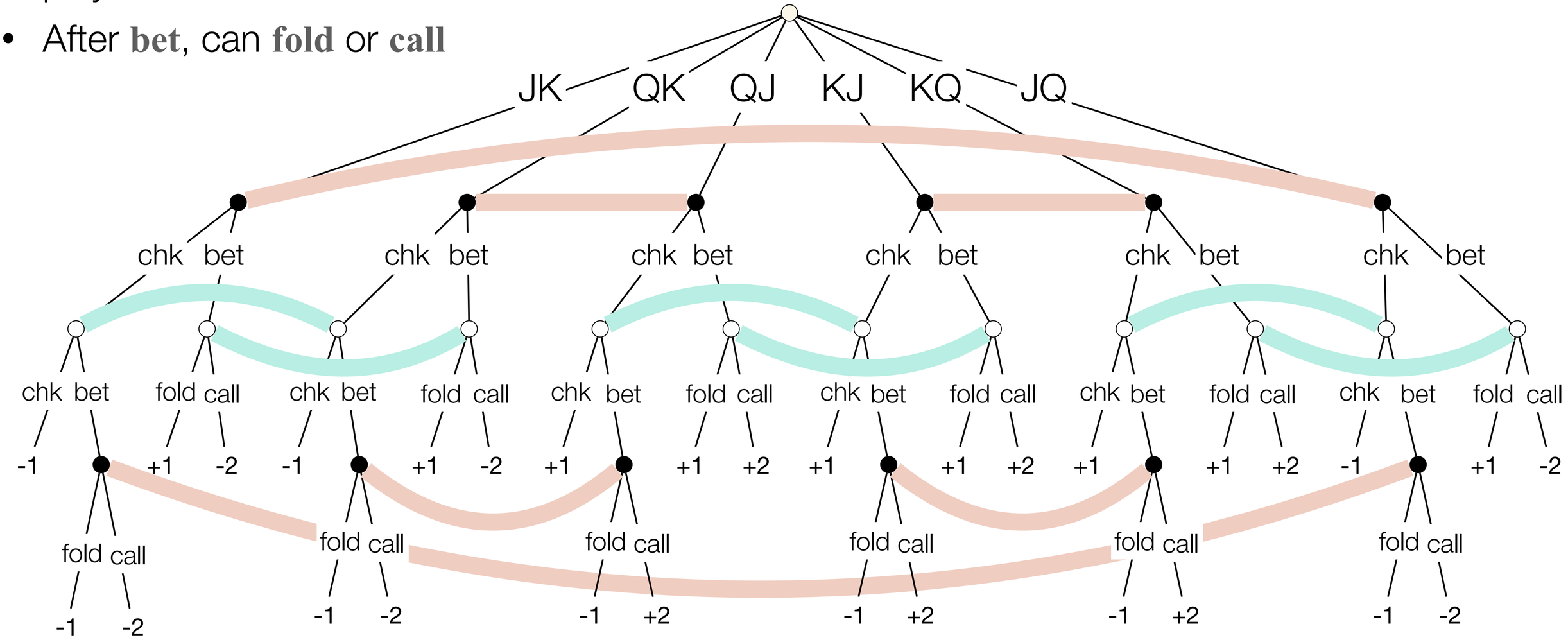


Hidden Random Information

- Many games have an element of **random information** that is revealed to only **some** of the players
- E.g., most card games:
  - The cards in my hand are **randomly assigned**
  - I know my **own** cards but not my **opponents**'
  - But I also know that they **don't have** any of the cards that **I have**
- This can be modelled as an extra player (**player 0** or **Nature**) whose actions represent the random occurence
  - e.g., the number that comes up on the die or the cards that are assigned
- **Nature** is assumed to be playing a **known**, **random** strategy

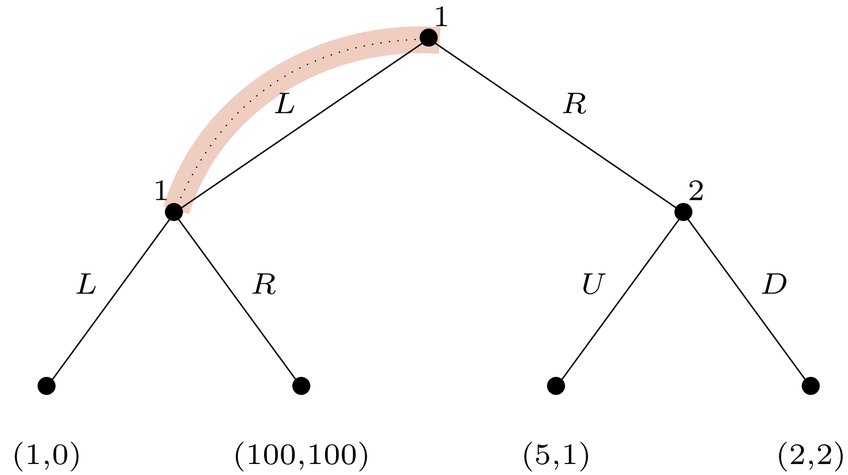
Example: Kuhn Poker

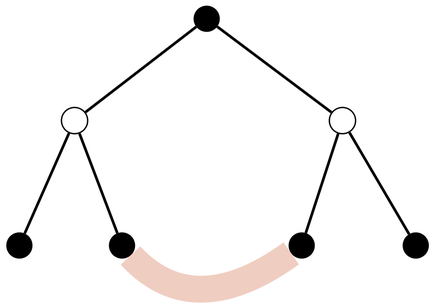
- 3 card deck, each player gets one card
- players can **check** or **bet**
- After **bet**, can **fold** or **call**

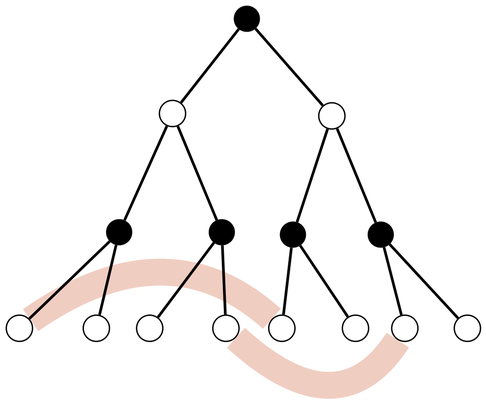


Perfect Recall vs Imperfect Recall

- A player has perfect recall in a game if they **never forget** information that they previously knew
- A player **forgets** information if they are no longer able to **distinguish histories** that they could previously distinguish
- *Examples:*
  - Forget **own actions**
  - Forget which **information sets** they were at before
- **Question:** Is **Kuhn poker** a game of **perfect recall** or **imperfect recall**?



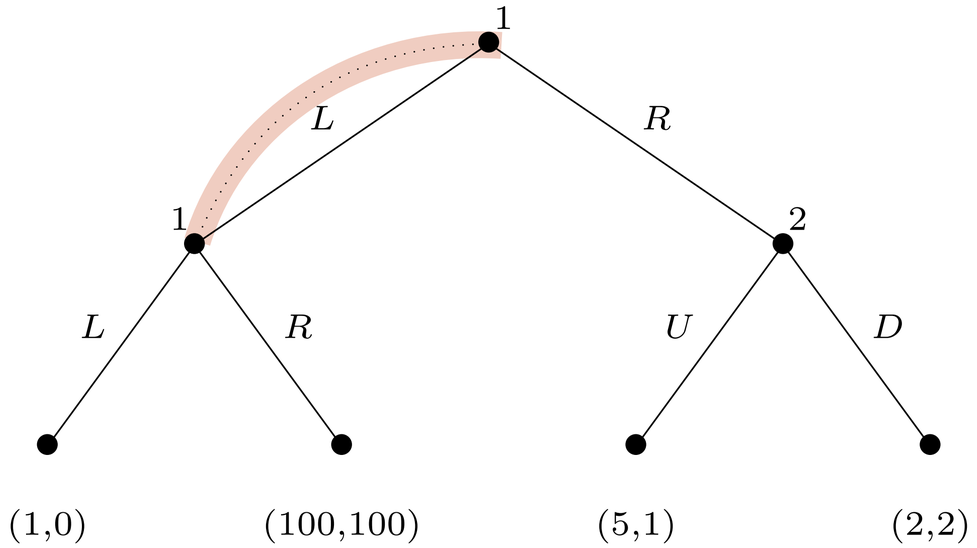




Mixed Strategies vs. Behavioral Strategies

Definition:
A **mixed strategy** is any distribution over an agent's **pure strategies**.

Definition:
A **behavioral strategy** is a probability distribution over an agent's actions at an **information set**, which is **sampled independently** each time the agent arrives at the information set.



Questions:

1. Can the outcome (100,100) be reached by any **mixed strategy**?
2. Can the outcome (100,100) be reached by any **behavioral strategy**?

Kuhn's Theorem

Theorem: [Kuhn, 1953]
In a game of **perfect recall**, any **mixed strategy** of a given agent can be replaced by an **equivalent behavioural strategy**, and any **behavioural strategy** can be replaced by an **equivalent mixed strategy**.

- Here, two strategies are **equivalent** when they induce the same probabilities on outcomes, for any fixed strategy profile (mixed or behavioural) of the other agents.

Corollary:
Restricting attention to behavioural strategies does not change the set of **Nash equilibria** in a game of **perfect recall**. (**why**?)

Summary

- **All** information is revealed in **perfect information games**; **no** information is revealed in **normal form games**
- Middle ground: **Imperfect information extensive form games**
- Players have **partial information** about which actions were played
  - **Information sets:** states that a player cannot distinguish
- **Pure strategies:** map from information set (not state) to action
  - **Mixed strategies:** randomly choose a **pure strategy** to play according to
  - **Behavioral strategies:** randomly choose an **action** at each **information set**
- **Kuhn's theorem: Mixed** and **behavioral** strategies equivalent in games of **perfect recall**