Mechanically Following AND/OR Strategies
&
Rock-Paper-Scissors

CMPUT 355: Games, Puzzles, and Algorithms

Lecture Outline

1. Logistics & Recap
2. Mechanically following AND/OR strategies
3. Rock-paper-scissors

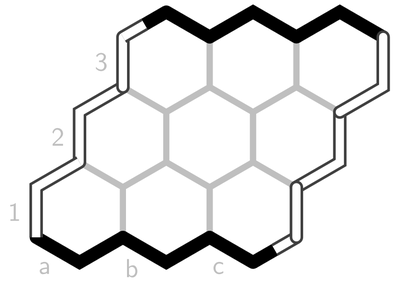
Logistics

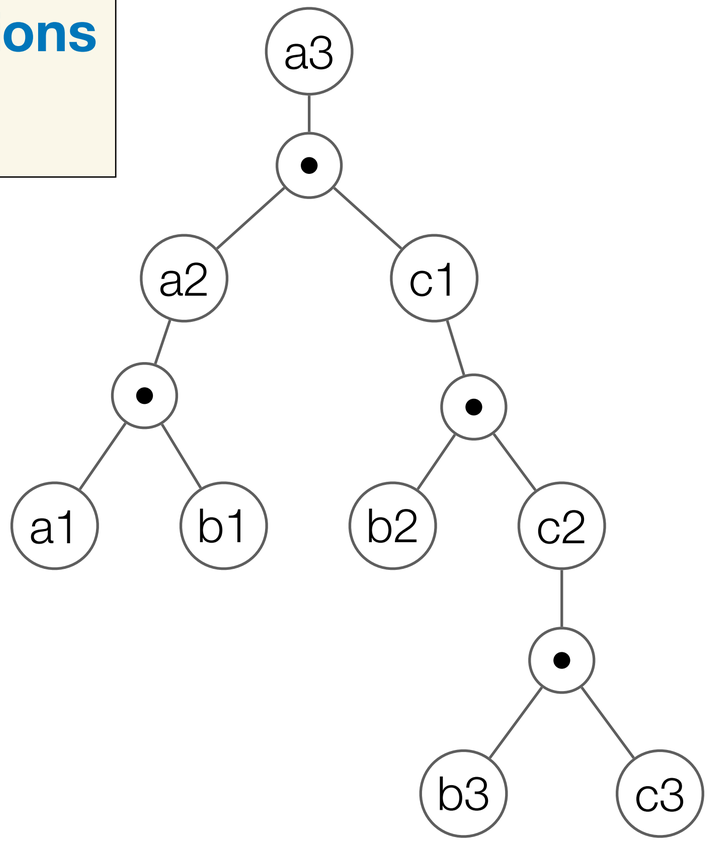
- **Practice questions #5** will be available by the end of **today** (Mar 20)
- **Quiz #5** is **next Friday** (Mar 27)
  - Covers up to the end of **today's lecture**
- **Quiz #3** marks have been posted
  - Solutions are available on the website
- **Quiz #4** marking is in progress (6/8 TAs are done!)

Recap: AND/OR Strategies

Definition:
An **AND/OR strategy** is either an **AND-expression** or an **OR-expression**, where:

- An AND-expression is a **cell location**, and 0 or more **OR-expressions**
- An OR-expression is 2 or more **AND-expressions**





We can write these multiple ways:

- $a3 \wedge ((a2 \wedge (a1 \vee b1)) \vee (c1 \wedge (b2 \vee c2 \wedge (b3 \vee c3))))$
- **a3. ((a2. (a1|b1)) | (c1. (b2 | (c2. (b3|c3)))))**

Executing AND/OR Strategies Mechanically

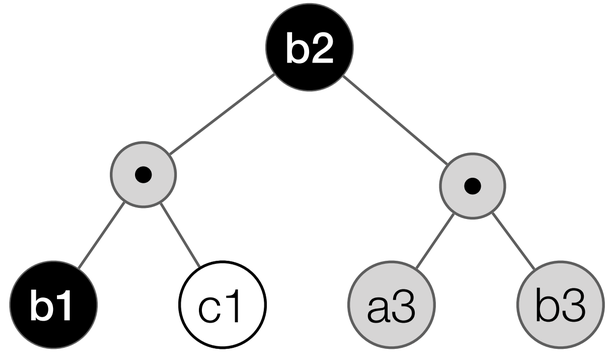
- Executing an AND/OR strategy means choosing a move that maintains an invariant: The opponent must not be able to make the logical expression false with a single move
- This can be **guaranteed** using the following mechanical procedure

- To execute an **AND strategy** that has a **single-cell** clause $\mathbf{x}$: **play** $\mathbf{x}$
- To execute an **AND strategy** with **multiple OR-clauses** after an opponent plays $\mathbf{x}$:
  1. Find OR-substrategy O_j that **contains** $\mathbf{x}$ in its carrier
  2. Find AND-substrategy A_k of O_j that **does not** contain $\mathbf{x}$ in its carrier
  3. Execute A_k

Example 1: Execution

To follow this strategy:

1. What must our first move be? (**why?**)
   - **1.B[b2]**
2. How should we reply to **1.B[b2] 2.W[c1]**?
   - Find OR-substrategy O_j that **contains** **c1**:
     - $(b1 \vee c1)$
   - Find AND-substrategy A_x of O_j that does **not** contain **c1**:
     - **b1**
   - Play A_x: **1.B[b2] 2.W[c1] 3.B[b1]**
3. How should we reply to **1.B[b2] 2.W[c1] 3.B[b1] 4.W[a2]**?



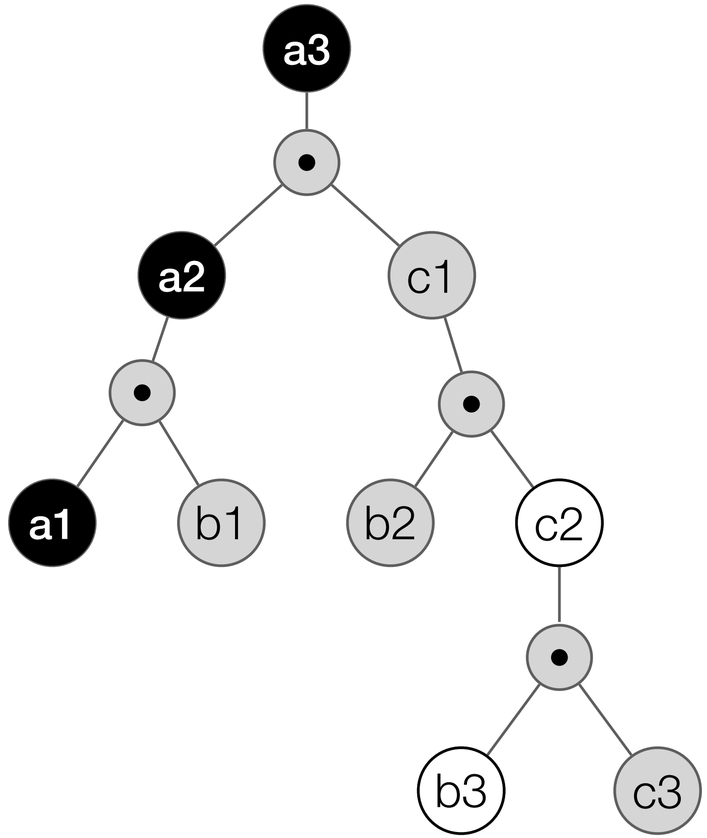
$$b2 \wedge (b1 \vee c1) \wedge (a3 \vee b3)$$

$$(b1 \vee c1) \wedge (a3 \vee b3)$$

$$(a3 \vee b3)$$

Example 2: Execution

1. What must our first move be? (**why?**)
   - **1.B[a3]**
2. How should we reply to **1.B[a3] 2.W[b3]**?
   - Find OR-substrategy O_j that contains **b3**:
     - $(a2 \wedge (a1 \vee b1)) \vee (c1 \wedge (b2 \vee (c2 \wedge (b3 \vee c3))))$
   - Find AND-substrategy A_x of O_j that does not contain **b3**:
     - $(a2 \wedge (a1 \vee b1))$
   - Play A_x: (**how?**)
     - **1.B[a3] 2.W[b3] 3.B[a2]**
3. How should we reply to **1.B[a3] 2.W[b3] 3.B[a2] 4.W[c2]**
   - Find OR-substrategy that contains **c2**
     - $a1 \vee b1 \vee (c1 \wedge (b2 \vee (c2 \wedge (b3 \vee c3))))$
   - Find AND-substrategy A_x of O_j that does not contain **c2**:
     - **a1**
   - Play A_x: **1.B[a3] 2.W[b3] 3.B[a2] 4.W[c1] 5.B[a1]**
   - **Question:** Is **a1** the **only** thing we could have played? (**why?**)



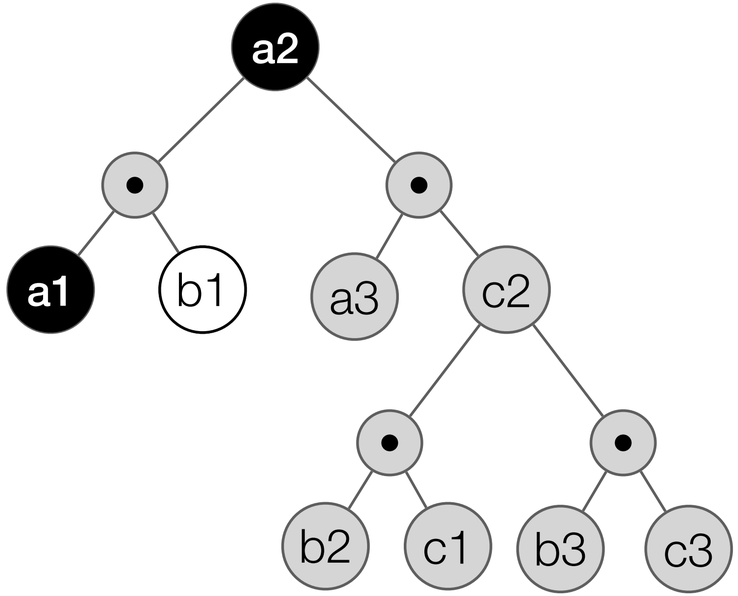
$$a3 \wedge ((a2 \wedge (a1 \vee b1)) \vee (c1 \wedge (b2 \vee (c2 \wedge (b3 \vee c3)))))$$

$$(a2 \wedge (a1 \vee b1)) \vee (c1 \wedge (b2 \vee (c2 \wedge (b3 \vee c3))))$$

$$a1 \vee b1 \vee (c1 \wedge (b2 \vee (c2 \wedge (b3 \vee c3))))$$

Example 3: Execution

1. What must our first move be?
   - **1.B[a2]**
2. How should we reply to **1.B[a2] 2.W[b1]**?
   - Find O_j that contains **b1**: $(a1 \vee b1)$
   - Find A_x of O_j that does not contain **b1:** $a1$
   - Play A_x: **1.B[a2] 2.W[b1] 3.B[a1]**
   - **Question:** Is **a1** the **only** thing we could have played? (**why?**)



$$a2 \wedge (a1 \vee b1) \wedge (a3 \vee (c2 \wedge (b2 \vee c1) \wedge (b3 \vee c3)))$$

$$(a1 \vee b1) \wedge (a3 \vee (c2 \wedge (b2 \vee c1) \wedge (b3 \vee c3)))$$

Rock-Paper-Scissors

- Two players (Row and Column)
- Players choose an action (**Rock**, **Paper**, or **Scissors**)
- Actions are revealed **simultaneously:**
  - Both chose **same**: tie
  - Rock beats Scissors
  - Scissors beats Paper
  - Paper beats Rock

	Rock	Paper	Scissor
Rock	0	-1	1
Paper	1	0	-1
Scissor	-1	1	0

Questions:

1. What does the **state graph** for this game look like?
2. What is the **optimal strategy** (against an optimal player)?

Normal Form Games

- Rock Paper Scissors is an example of a **normal form game**
- Players choose actions **simultaneously**
- Each possible **combination** of actions maps to scores for the players ("**utilities**")
- To specify a normal form game, list:
  - **Who** the players are (need not be just 2)
  - What **actions** each player has available (need not be the same per player)
  - The **utility** to each player for each combination of actions
    (often a table for 2-player games)
- Many **normal form games cannot** be represented as a **state graph** (**why?**)
- **Every state graph** game **can** be represented as a **normal form game**! (**how?**)
  - (This is actually why they are called normal form games)

	Rock	Paper	Scissor
Rock	0,0	-1,1	1,-1
Paper	1,-1	0,0	-1,1
Scissor	-1,1	1,-1	0,0

Mixed Strategies

- Every strategy (action) in Rock Paper Scissors can be defeated by another!
- So neither player has an **optimal deterministic strategy**
  - No matter what we choose, at least one player would have been better off choosing something else
  - E.g., if we both chose **Paper**, I would have been better off choosing **Scissors**
  - If I chose **Scissors** and you chose **Rock**, I would have been better off choosing **Paper**
- In a **normal form game**, optimal strategy is sometimes **randomized** (i.e., "**mixed**")
  - In those cases, an optimal strategy has maximal **expected** score against optimal opponent(s)
- In a **perfect information alternating moves** game (e.g., Nim, Hex, Tic Tac Toe), there is **always** a **deterministic** optimal strategy (**why?**)

Summary

- **AND/OR strategies** can be executed **mechanically**:
  - First move plays the singleton cell in the top-level AND strategy
  - Reply to a move: Find the largest enclosing OR-strategy for the opponent's move
    - Play the AND-strategy in a different branch of the OR-strategy
    - If opponent's move is not in the strategy, play arbitrarily
- In **normal-form games**, players choose a single move **simultaneously**
  - Example: Rock Paper Scissors
  - Optimal strategies are sometimes **mixed** (i.e., randomized)
    - Alternating-move perfect information games always have deterministic optimal strategies
  - Every alternating-move perfect information game can be written as a normal-form game
    - But not necessarily the other way around