Compact Hex Strategies

CMPUT 355: Games, Puzzles, and Algorithms

Lecture Outline

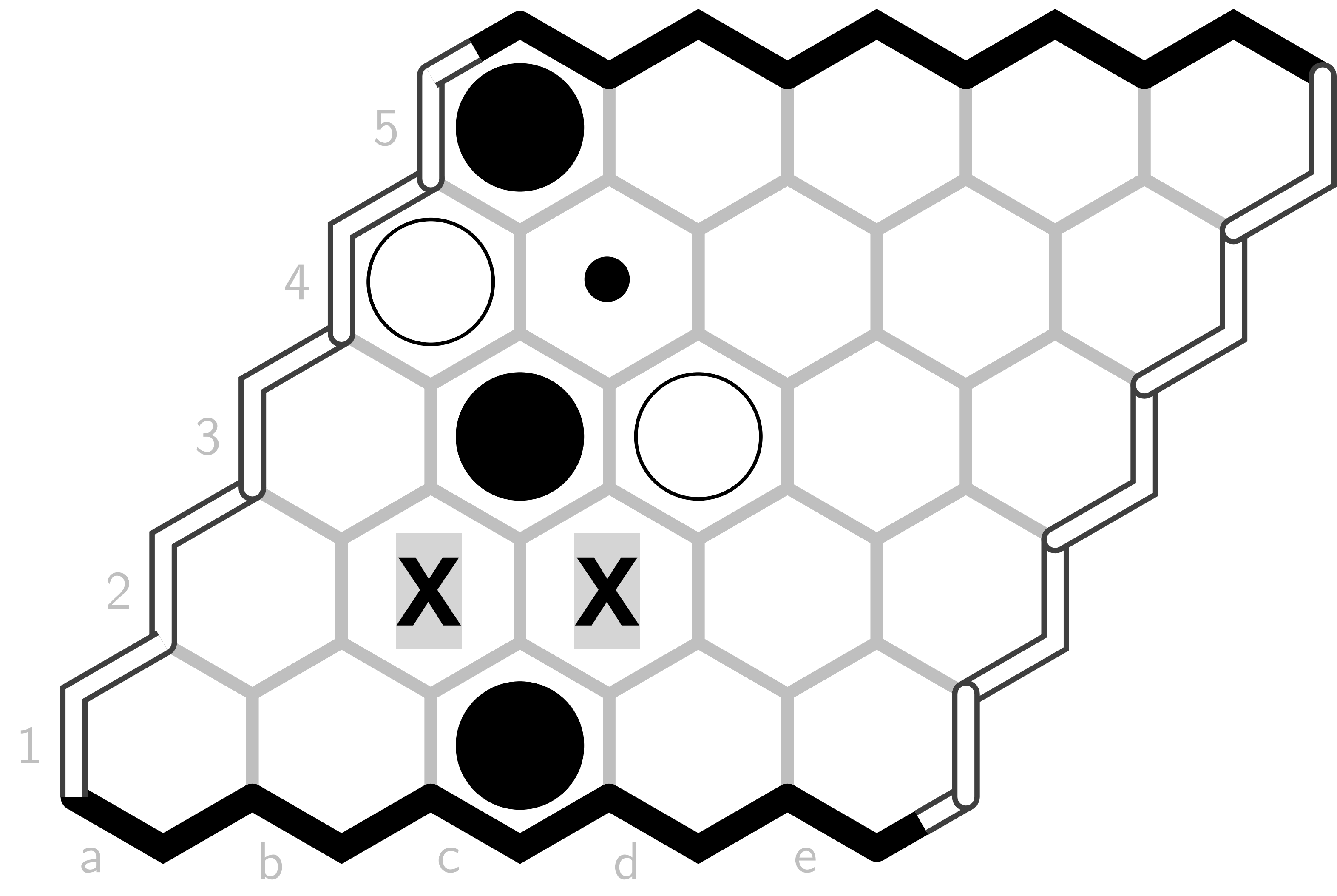
1. Logistics & Recap
2. Spider diagrams
3. AND/OR strategies

Logistics

- **Practice questions #4** will be released today
  - Solutions will be released on Tuesday (Mar 10)
- **Quiz 4** is **next Friday** (Mar 13)
  - *Coverage:* up to and including **this lecture**
  - Bring your student ID!
  - No calculators or other devices
  - The quiz will be run by 3 TAs (I will be in the hospital)

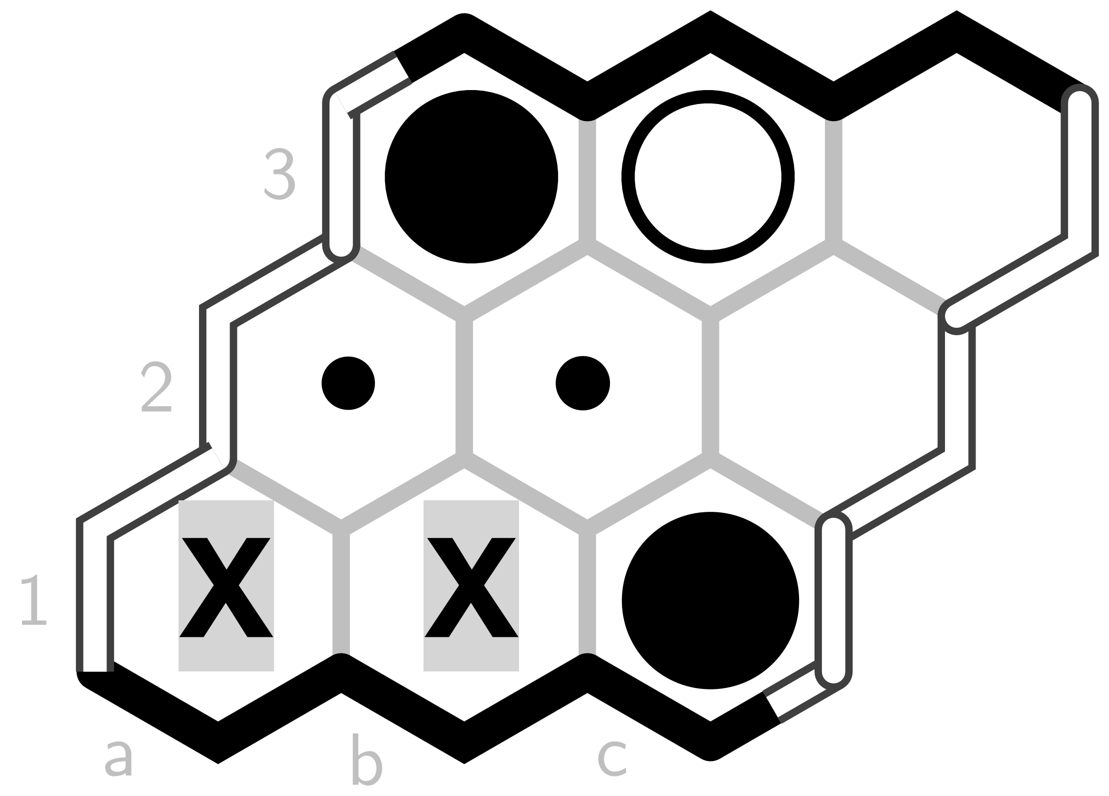
Recap: Virtual Connections

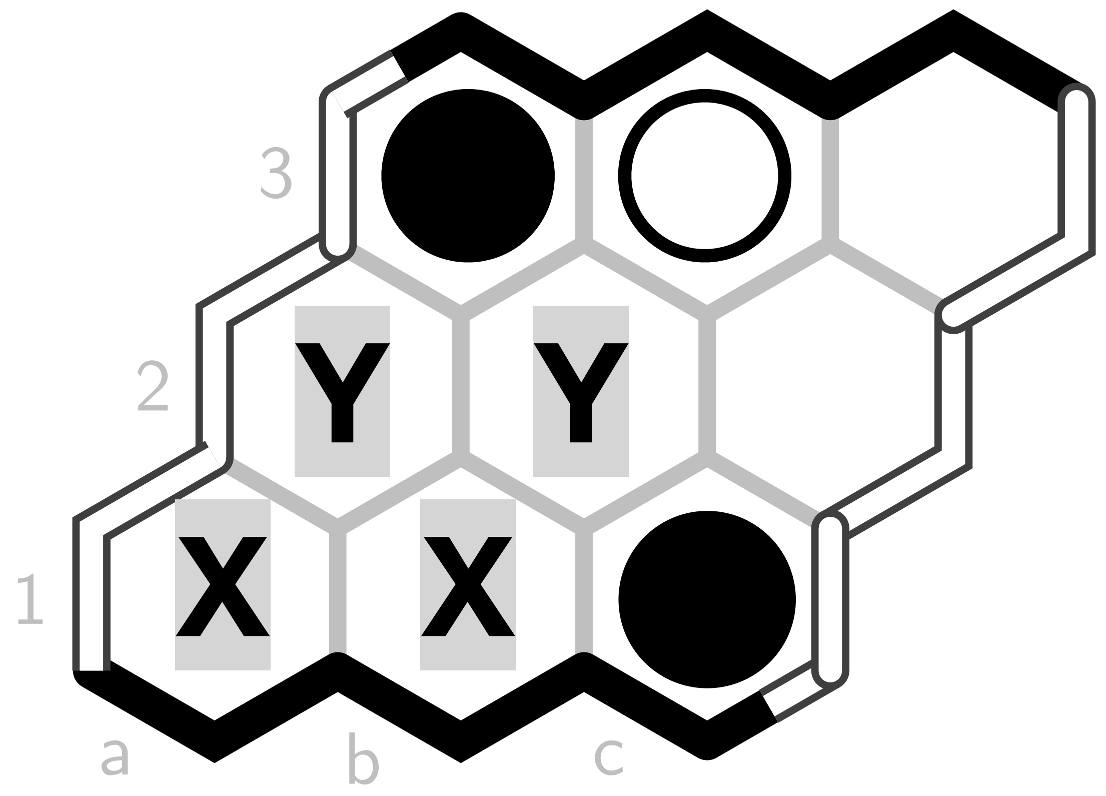
- **Virtual connection:**
  A player **can** connect two cells, but they haven't **yet**
- **Semi connection:** A player can force a connection between two cells, but only if they are the **current** player to move
  - **Example:** Black can force a connection between **b3** and **a5**, **if it is Black's turn**.
- **Full connection:** A player can connect two cells whether they are the **current** player to move *or* the **next** player to move.
  - **Example:** Black can force a connection from **b3** to **c1** edge **even if it's White's turn**.
  - Two nodes are fully connected if they have **2 or more semi-connections**



Recap: Pairing Strategies

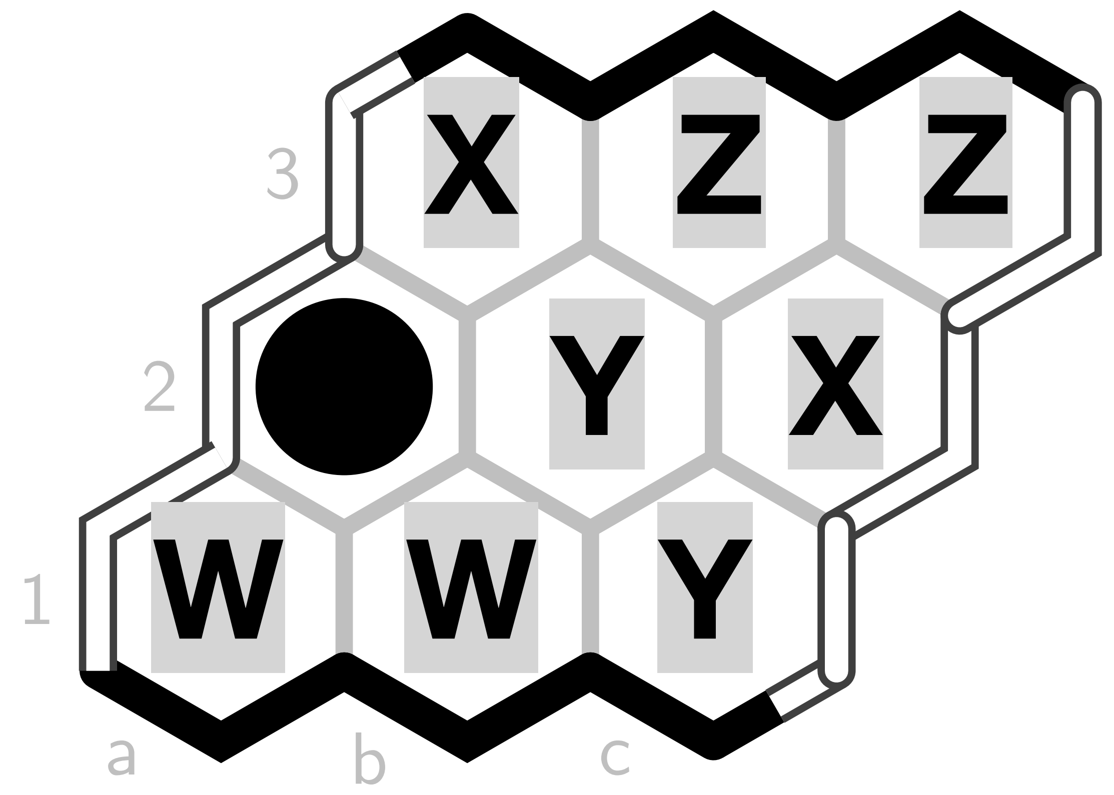
- Pairing strategy:
  - Group empty cells into **pairs**
  - Whenever opponent plays one cell of a pair, reply on the **other cell**
- **Example:**
  - If White plays one of {**a2**, **b2**}, play **the other one**
  - If White plays one of {**a1**, **b1**}, play the other one
- **Easy to execute:**
  - Search for pair that opponent played on, play on the other
  - If no pair found, play on an arbitrary pair (or arbitrary unlisted cell!)
- **Compact:** Don't need to store a contingency for every possible opponent move
- But: **not every winning strategy** can be represented as a pairing strategy
  - Even for those that can, it can be **hard to derive**

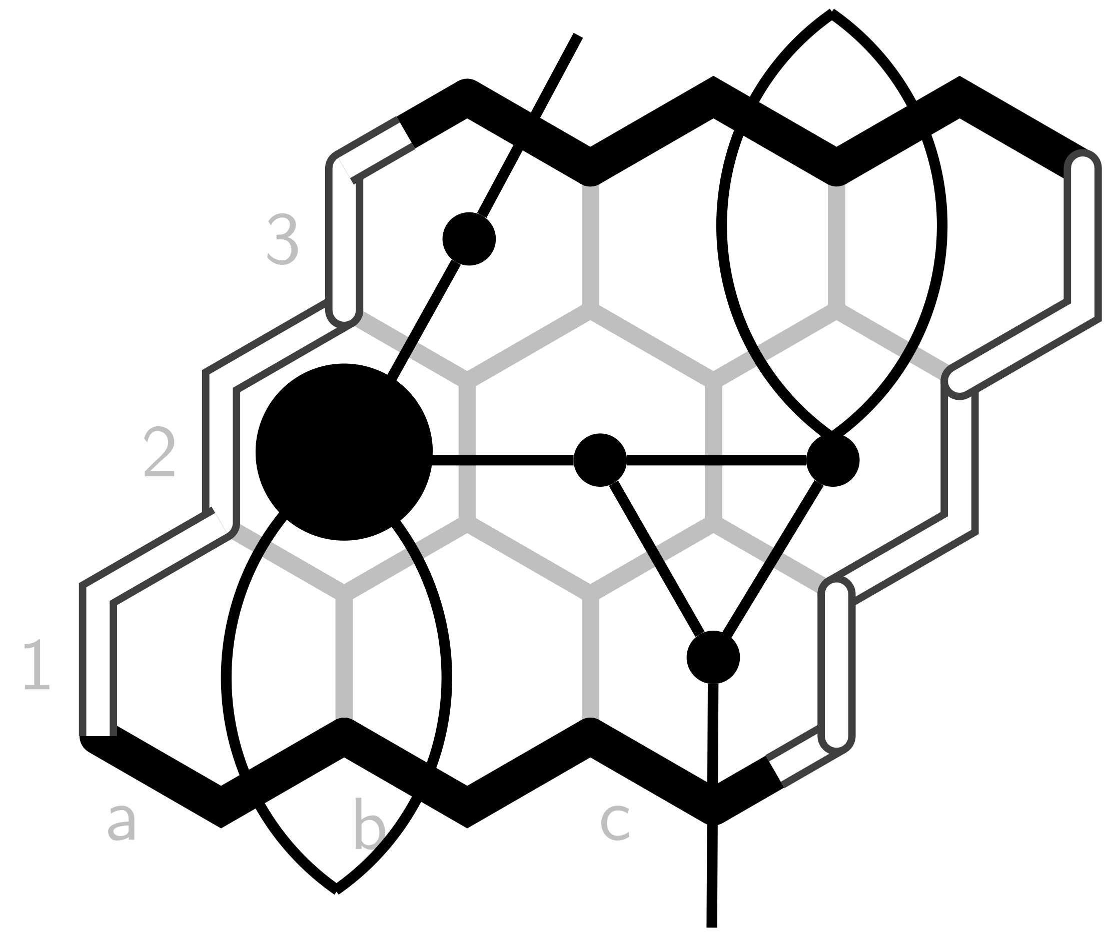




Spider Diagrams

- A **spider diagram** lets you represent the combination of virtual connections visually:
    - single line with dot for semi-connections
    - double lines for full connections
- Example: we argued for the pairing strategy at right using virtual connections, but it's hard to see them in the pairing representation
- Drawback: how do you play this strategy?

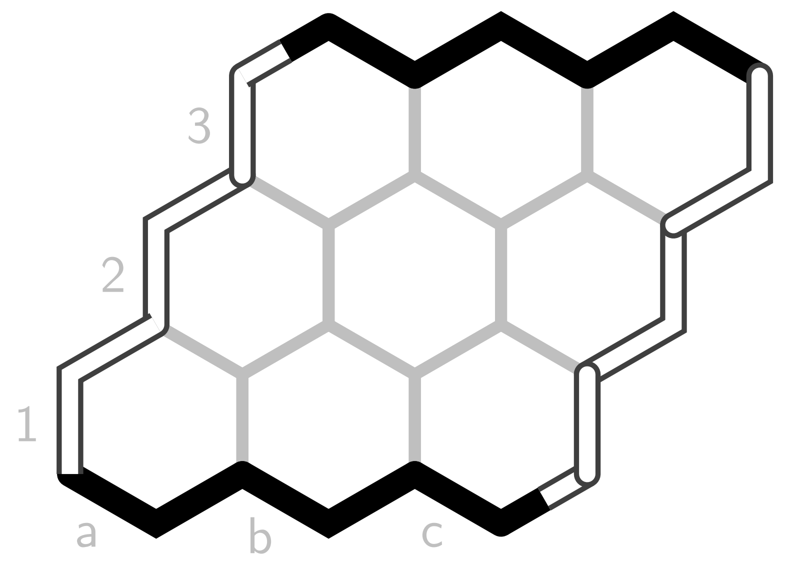


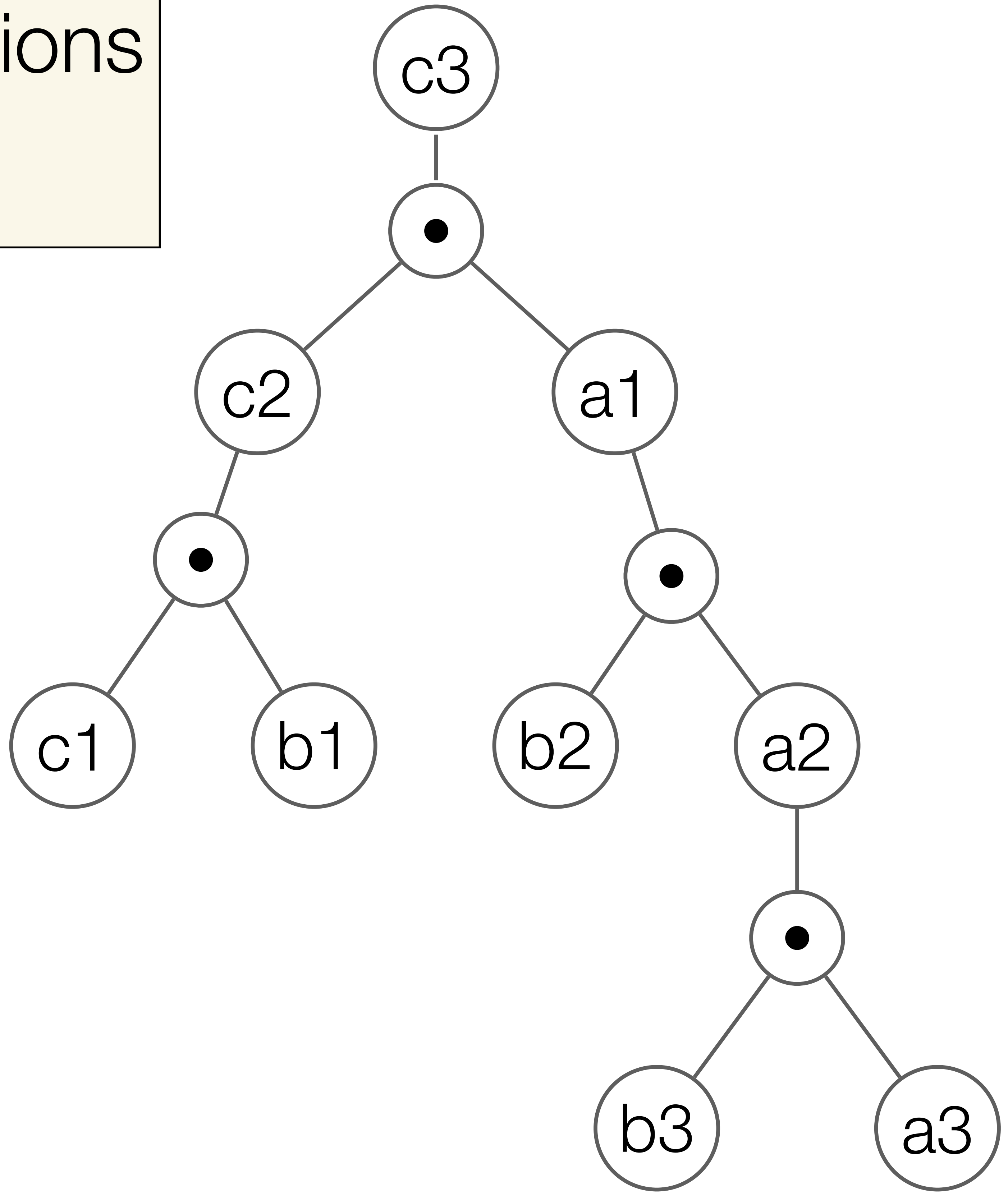


AND/OR Strategies

Definition:
An AND/OR strategy is either an AND-expression or an OR-expression, where:

- An AND-expression is a cell location, and 0 or more OR-expressions
- An OR-expression is 2 or more AND-expressions



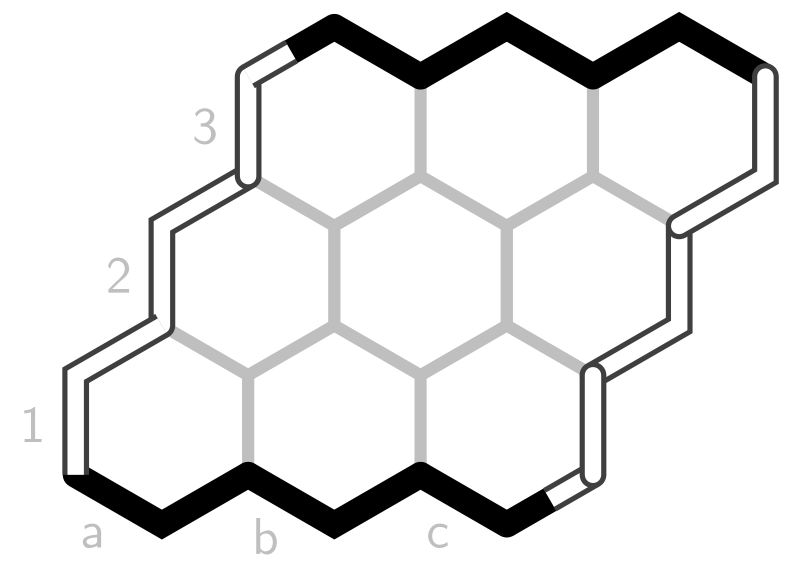


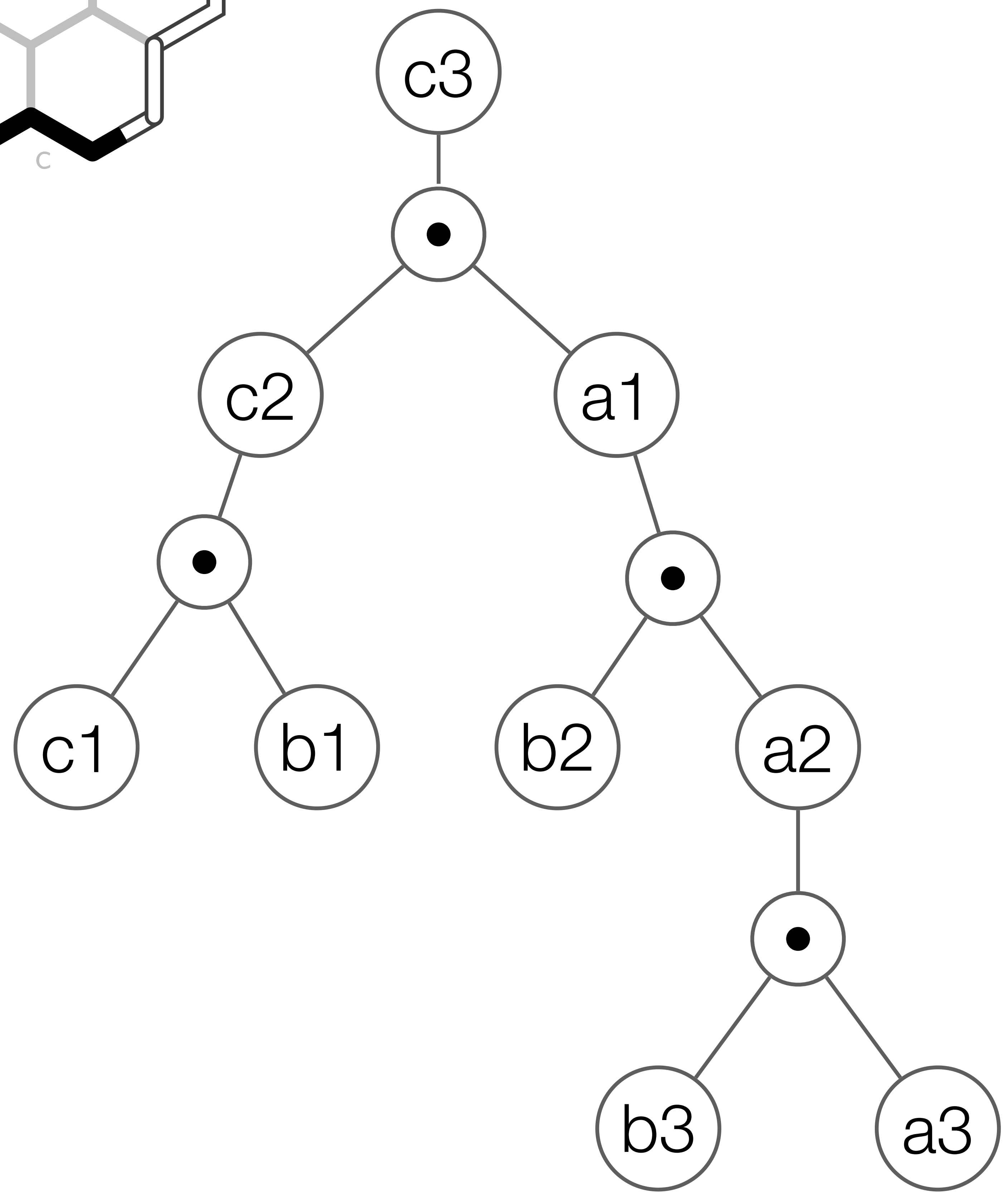
We can write these multiple ways:

- $c3 \wedge ((c2 \wedge (c1 \vee b1)) \vee (a1 \wedge (b2 \vee a2 \wedge (b3 \vee a3))))$
- **c3. ((c2. (c1|b1)) | (a1. (b2 | (a2. (b3|a3)))))**

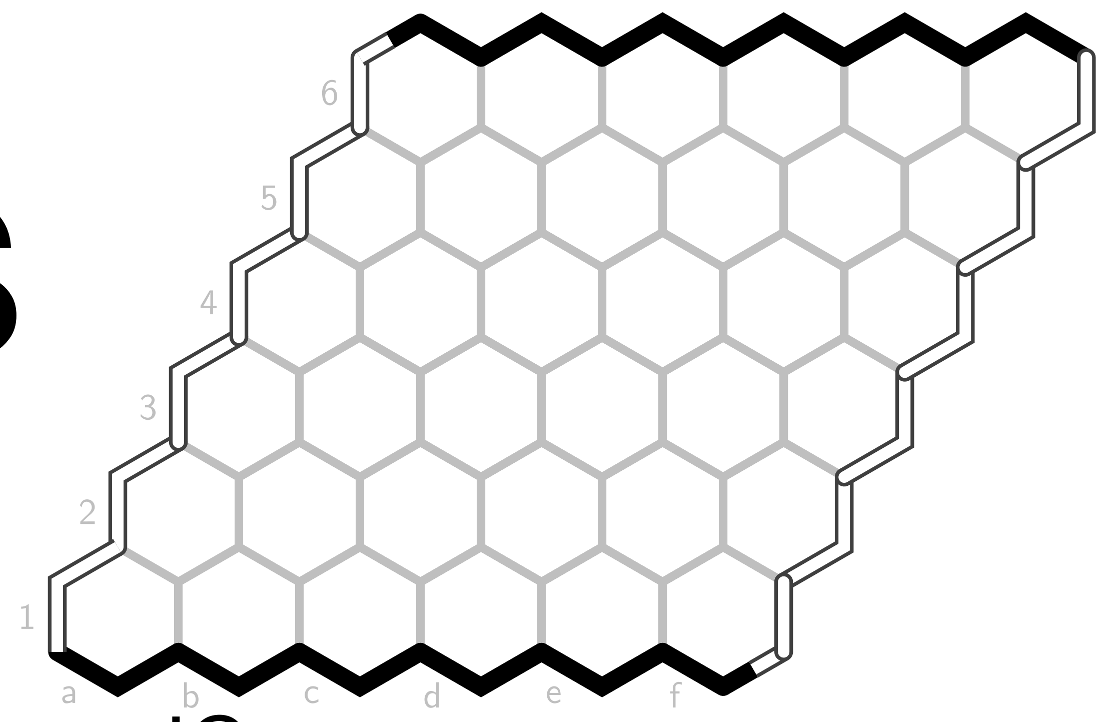
Executing AND/OR Strategy

- Like a pairing strategy, AND/OR strategies do not need to be executed in order!
- Strategies for the current player are AND expressions; strategies for the next player are OR expressions.
- Play to force the expression to be true:
  - i.e., if there is a cell that would make the expression false, we must play on that cell
  - e.g.: on our first turn, we **must** play **c3** (**why?**)
- If the opponent makes one branch of an OR expression false, we must play in the other branch
  - e.g., if the opponent plays on **c2**, we **must** play on **a1**
- **Question:** If we play on **c3** and then opponent plays on **b1**, where **must** we play?





Compactness Analysis



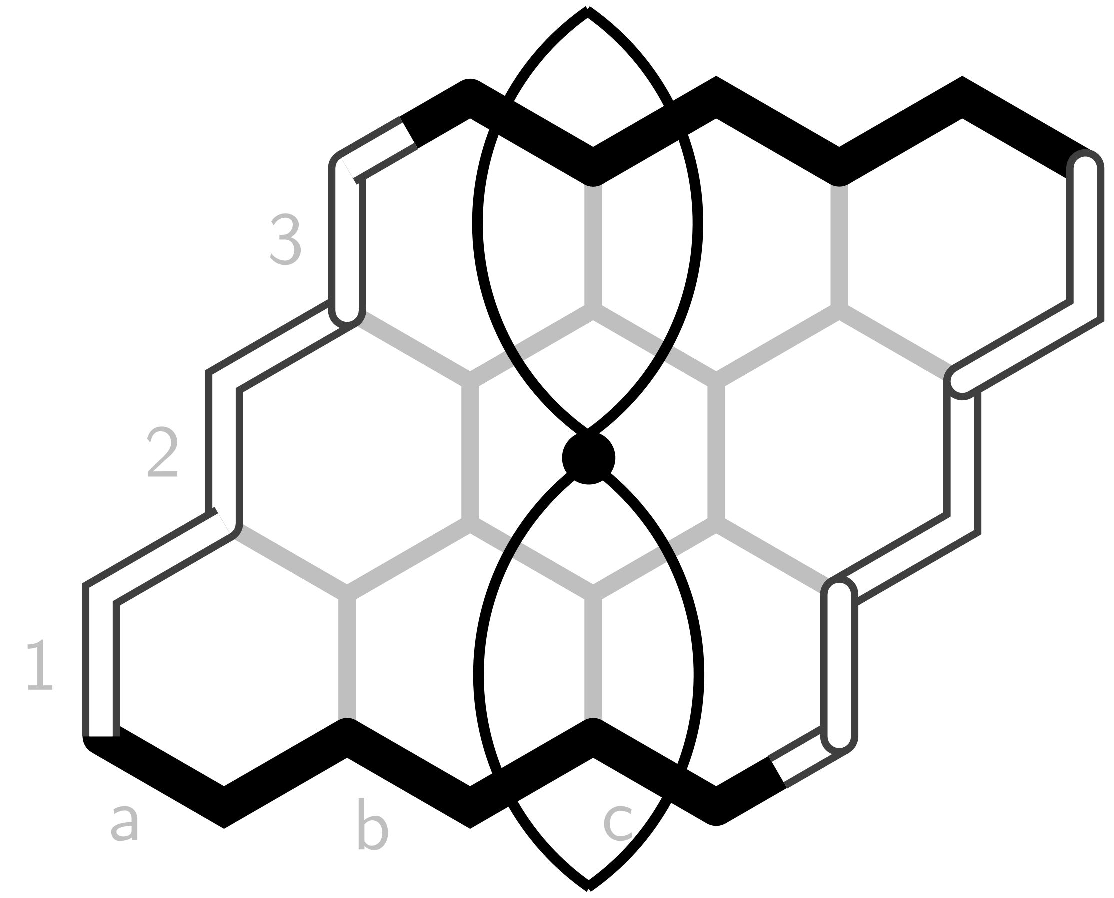
- **Question:** How many possible games of Hex are there on a 6×6 board?
  - It turns out most games are done by the time the board is about half full
- We know that the empty position is a winning position for Black (**why?**)
- **Question:** How many nodes would a proof tree have?
  - Every Black move has **every possible** White move
  - Every White move has **one** Black move
- Something like $1 \times 35 \times 1 \times 33 \times \cdots \times 19 \times 1 = 6{,}432{,}002{,}143{,}875$ nodes!
- AND/OR strategies are dramatically more compact (**why?**)
  - Don't need to represent the **opponent moves** at all ("opponent oblivious")
  - Don't need to account for the **order** of the moves

Cell Sets

- An AND/OR strategy describes a condition which, if true, leads to a win
  - The cell locations are set to **true** if the cell contains a stone of the player's colour
- For any given expression, a finite set of cell combinations that will make it true
- These are the **cell sets** of the strategy

Questions:

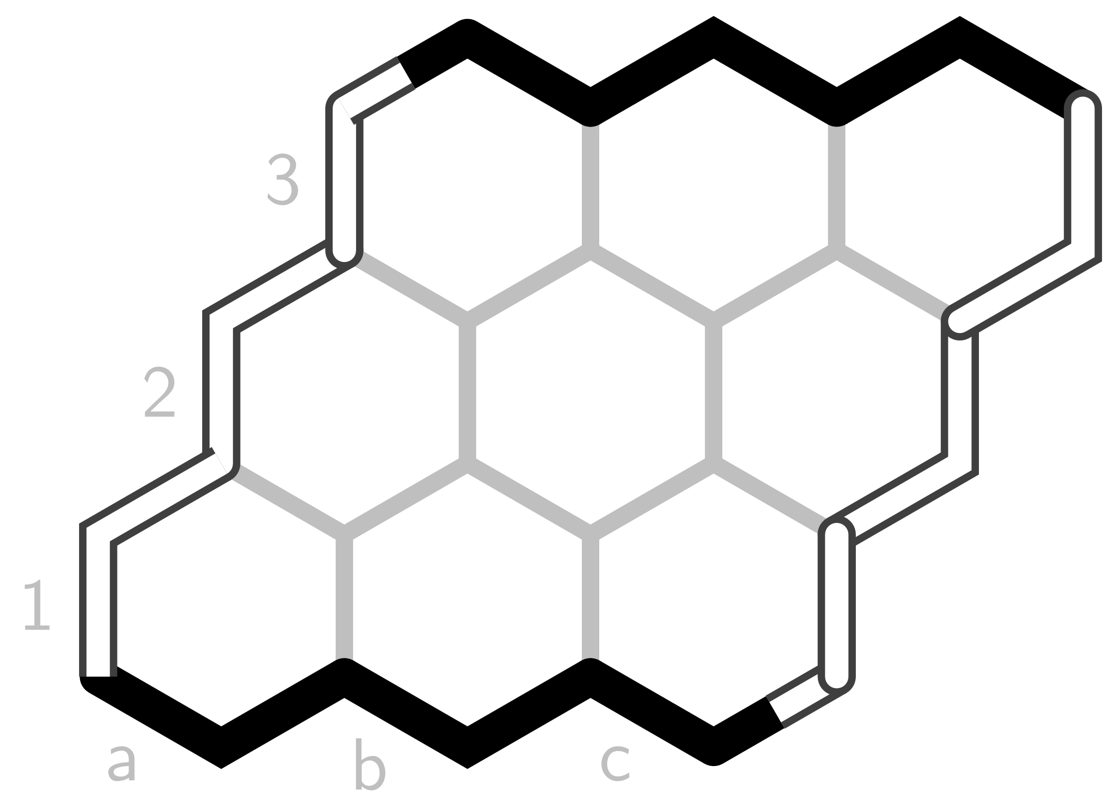
1. **How many** cell sets does this AND/OR strategy have? (**why?**)
2. What are they?



$$b2 \wedge (b1 \vee c1) \wedge (a3 \vee b3)$$

Winning AND/OR Strategies

- **Not every** AND/OR strategy is a winning strategy
- E.g., $b1 \wedge (a2 \vee b2) \wedge (b3 \vee c3)$ does not guarantee a win (**why?**)
- To be a winning strategy:
  1. **Every** cell set of the expression must link the two borders
  2. For each OR expression, the cell sets of the **subclauses** must be **disjoint** (**why?**)



Every Winning Strategy Can Be AND/OR

In Hex, **every** winning strategy from an arbitrary position can be represented as an AND/OR strategy!

- A winning strategy is a **full** or **semi-connection** from one border to the other (**why?**)
- A **semi-connection** is a **single move**, possibly combined with some full connections
- A **full connection** is a set of **2 or more semi-connections**, with empty intersections
- Recursively:
  - Every single move can be written as an AND expression with no OR-subexpressions
  - Every full connection can be written as an OR-expression combining the semi-expressions
  - Every semi-connection can be written as an AND-expression combining a single move with a set of (optional) full connections

Summary

- We know that 6×6 Hex is a win for Black
- *But,* 6×6 Hex has a large enough state space that it is infeasible to write down a proof tree for a winning strategy
- **AND/OR strategies** give a dramatically more compact way to represent strategies
  - Correspond to a recursive **semi-connection** (if a **current** player win) or **full connection** (if a **next** player win)
- AND/OR strategies are **winning** if:
  1. Every cell set links the two opposite borders
  2. The cell sets of each subexpression of an OR-expression are disjoint