Alpha-Beta Search

CMPUT 355: Games, Puzzles, and Algorithms

Lecture Outline

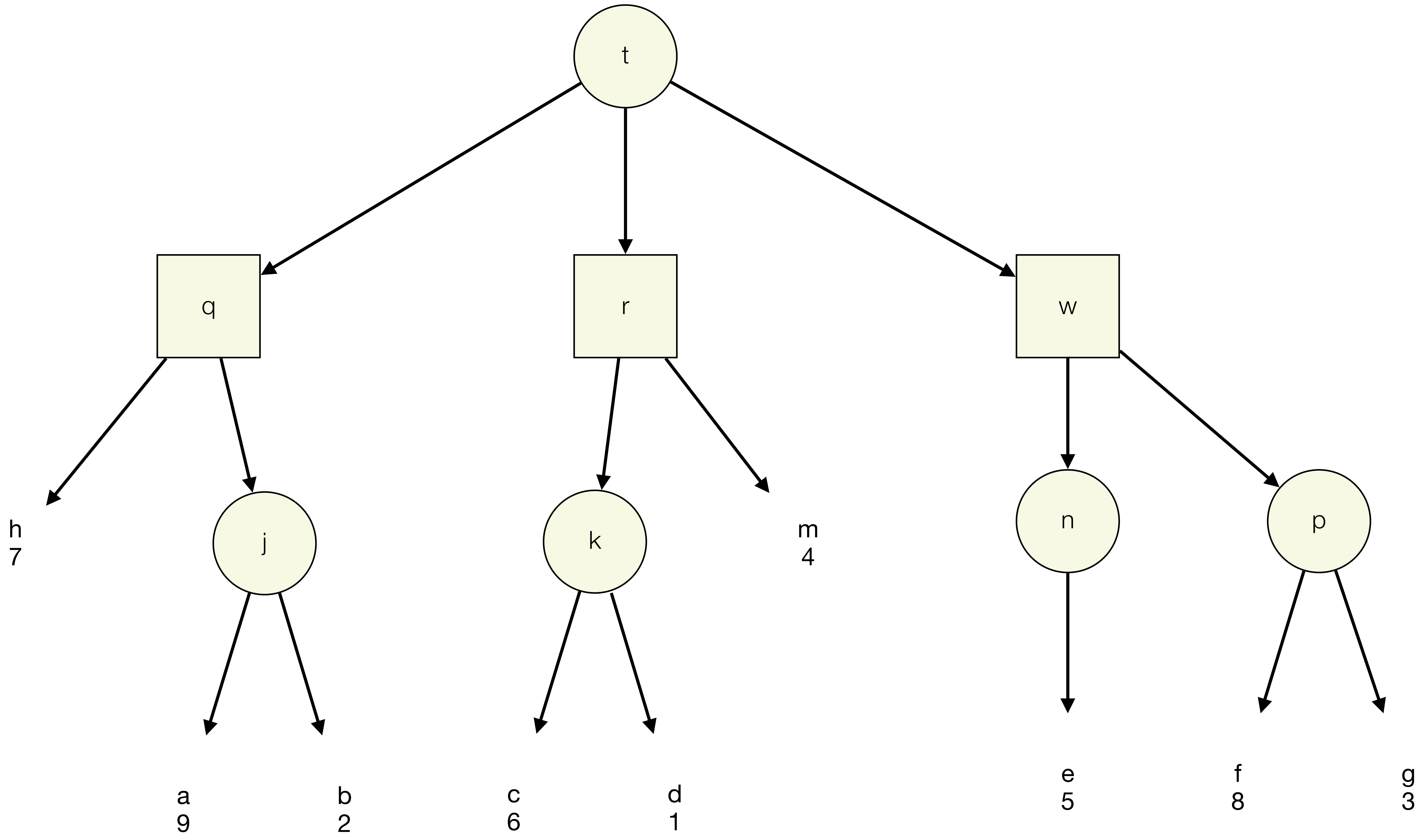
1. Logistics & Recap
2. Pruning
3. Alpha-beta search
4. Examples

Logistics

- **Practice quiz questions #2:**
    - Now available
    - Answers posted yesterday
- **Quiz 1 marks available** (average: approximately 14/20)
    - Solutions have been posted
    - Scans are on Canvas
    - Any concerns about grading should be raised in a comment on the Canvas submission
- **Quiz 2:** Friday, **Feb 6**
    - In-class, full 50 minutes
    - **No need to email** if you have to miss it;
      up to 3 missed quizzes replaced by final exam **automatically**
    - Coverage: up to the end of **last Friday's lecture** (Sliding tiles & subgoals)
    - Questions will be very similar to practice questions

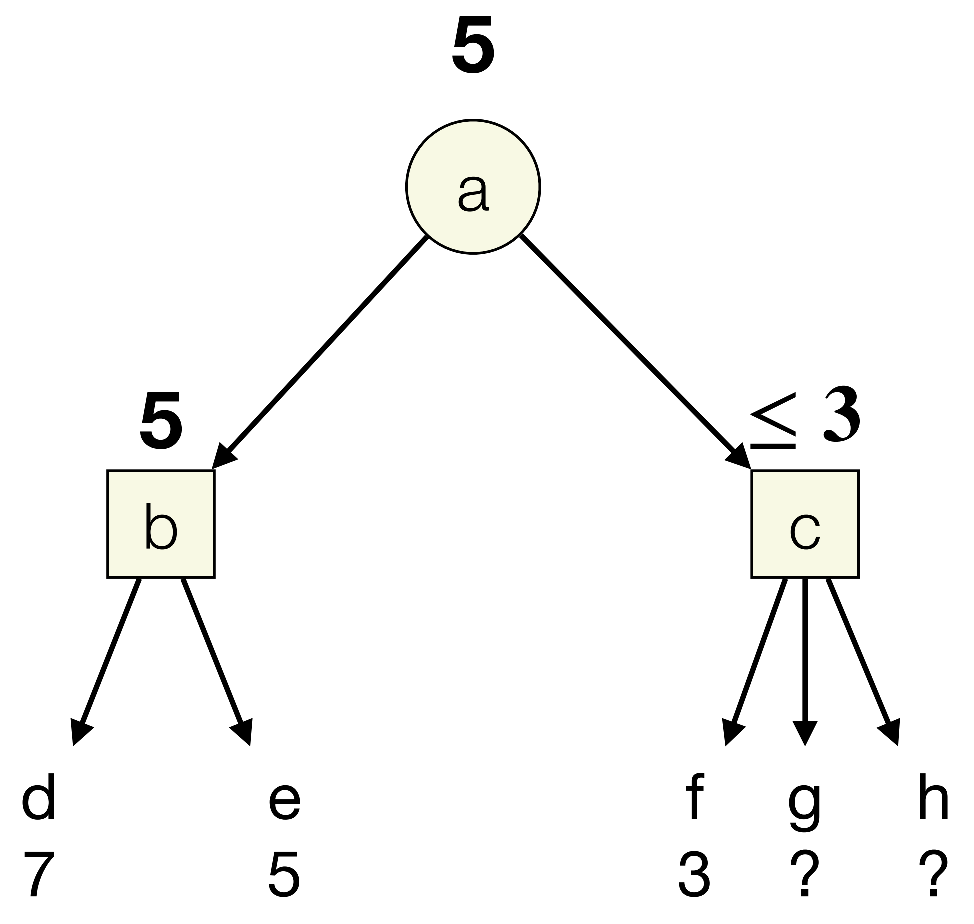
Recap: Minimax Search

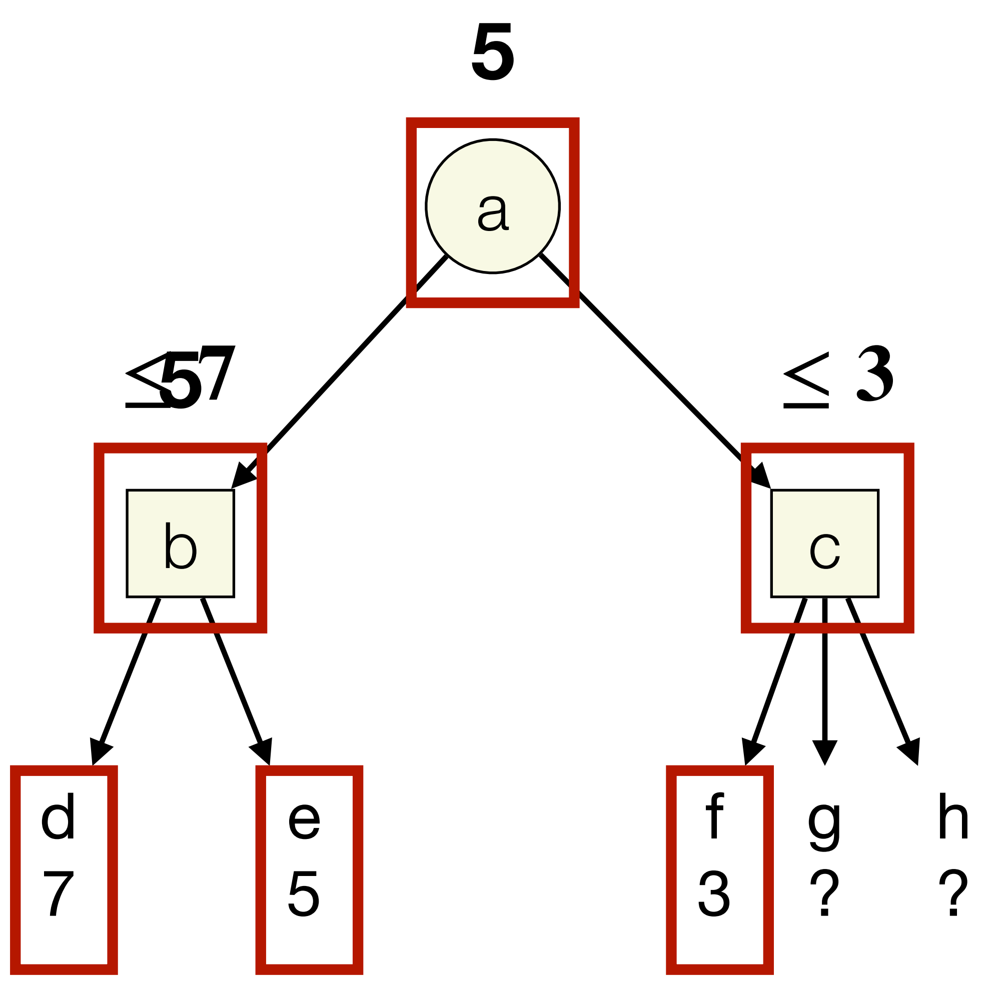
```
assume P1 plays at root
assume players alternate turns

def score(s):
    return P1's score at state s

def minimax(s):
    if terminal(s):
        return score(s)
    if player(s) == 1:
        return max{minimax(c) for
all c in children(s)}
    if player(s) == 2:
        return min{minimax(c) for
all c in children(s)}
```



Pruning

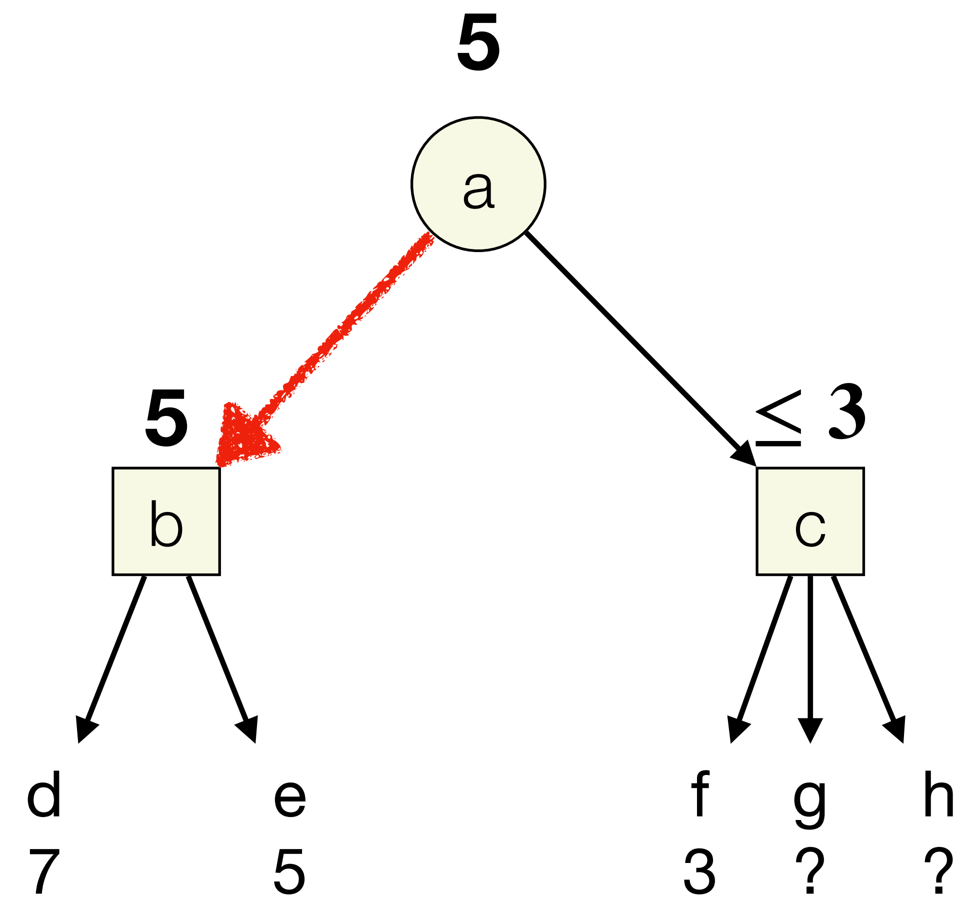
- Plain minimax search visits every node in depth-first order
- But it's sometimes possible to avoid searching some subtrees based on information partway through the algorithm
- Once we have explored the b's subtree and the first child of c, we know:
    - minimax value of b is 5
    - minimax value of c is **no larger than 3** (**why?**)
- But a is a max node!
    - **No matter what** minimax value of c turns out to be, P1 will choose b instead (**why?**)
    - So minimax value of a is 5
    - ...even though we haven't explored g or h yet
    - those nodes were **pruned**
- **Alpha-beta search** checks as it goes for these pruning opportunities





Alpha-Beta Search, informally

- **Main idea:** Learn **enough** about the current node guarantee that minimax strategies **never play here**
- What can guarantee that?
  1. Current node is a **max** node, *and*
     minimax value of current node is **at least α**, *and*
     a **min** node **earlier in the tree** has a choice that guarantees **less than α**
  2. Current node is a min node, *and*
     minimax value of current node is **less than β**, *and*
     a **max** node **earlier in the tree** has a choice that guarantees **more than β**
- **Question:** Why is earlier in the tree important?
- At each stage, alpha-beta search tracks:
  - α: **Highest** value available on **path from current node to root** (including both current node and root)
  - β: **Lowest** value available on **path from current node to root** (including both current node and root)



Alpha-Beta Search Pseudocode

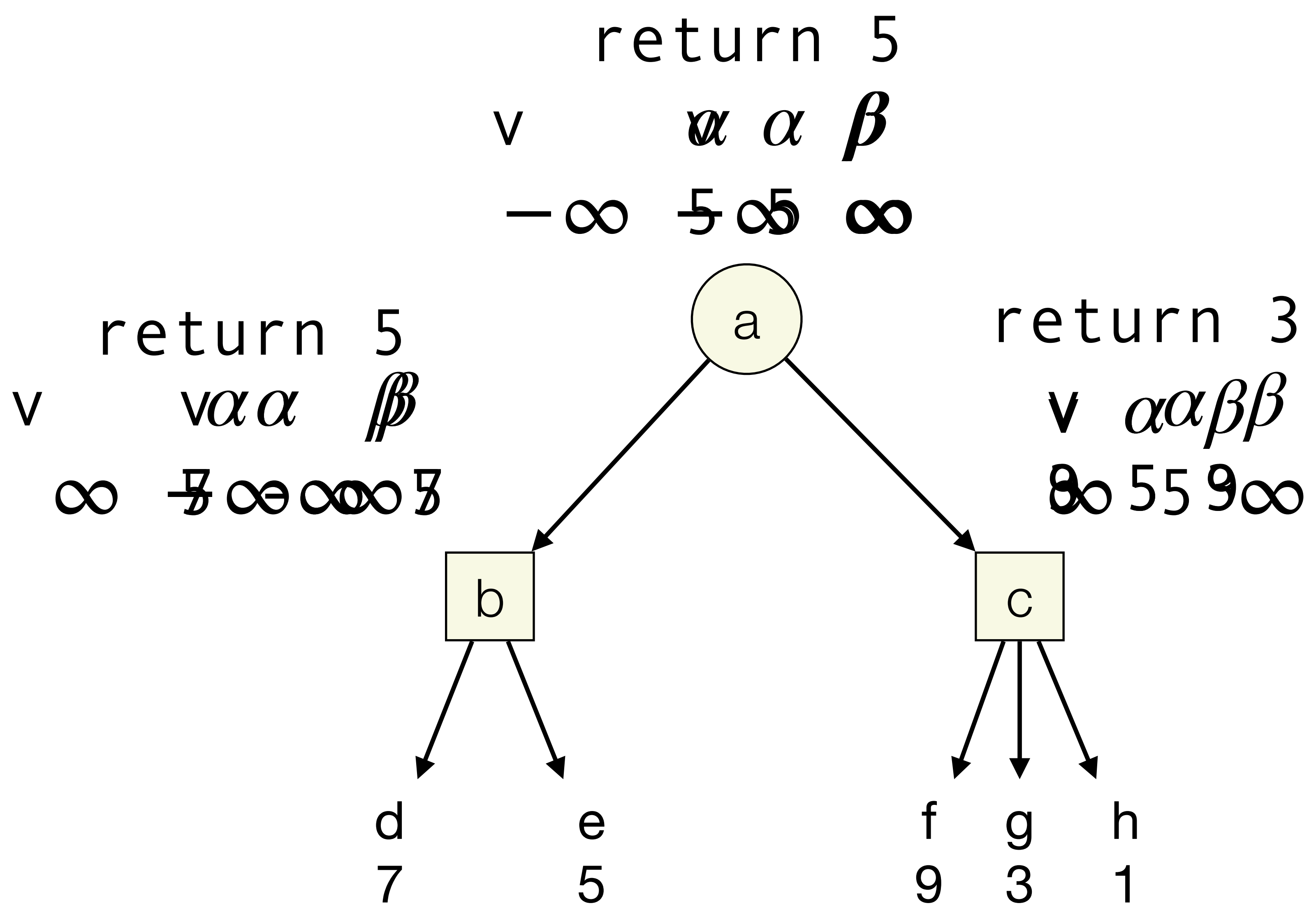
```
def alphabeta(s, alpha, beta):
 if terminal(s):
  return score(s)
 if player(s) = 1: # MAX player
  val = -inf
  for c in children(s):
   ab = alphabeta(c, alpha, beta)
   alpha = max(alpha, ab)
   val = max(val, ab)
   if alpha >= beta:
    return val # prune remaining children
 if player(s) = 2:
  val = inf
  for c in children(s):
   ab = alphabeta(c, alpha, beta)
   beta = min(beta, ab)
   val = min(val, ab)
   if alpha >= beta:
    return val # prune remaining children
 return val
```

Questions:

1. Why do we initialize `val` with `inf` or `-inf`?
2. With what arguments should we call `alphabeta` on the root node? (**why?**)
3. Why do we not return updated `alpha` / `beta` values as well as `val`?

Alpha-Beta Search Example

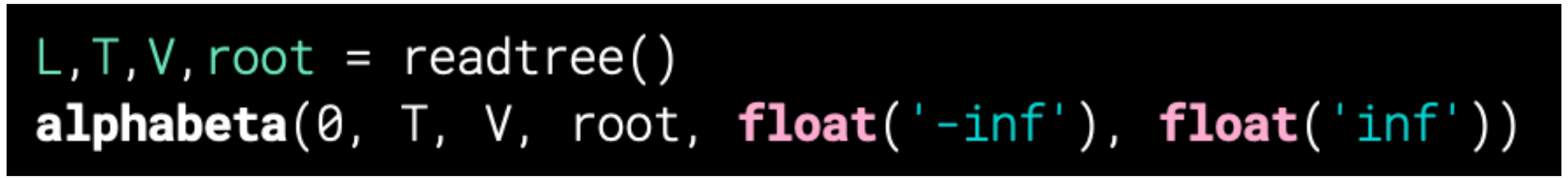
```
def alphabeta(s, alpha, beta):
  if terminal(s):
    return score(s)
  if player(s) = 1: # MAX player
    val = -inf
    for c in children(s):
      ab = alphabeta(c, alpha, beta)
      alpha = max(alpha, ab)
      val = max(val, ab)
      if alpha >= beta:
        return val # prune remaining children
  if player(s) = 2:
    val = inf
    for c in children(s):
      ab = alphabeta(c, alpha, beta)
      beta = min(beta, ab)
      val = min(val, ab)
      if alpha >= beta:
        return val # prune remaining children
  return val
```



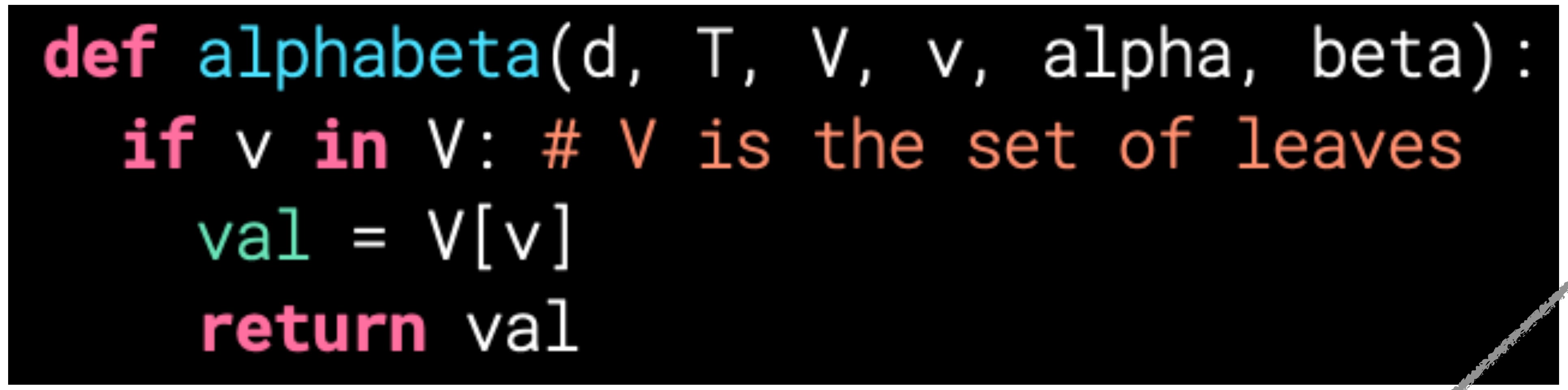
Demo:

```
% cd abeta/
% python3 alphabeta.py < t1.in
```

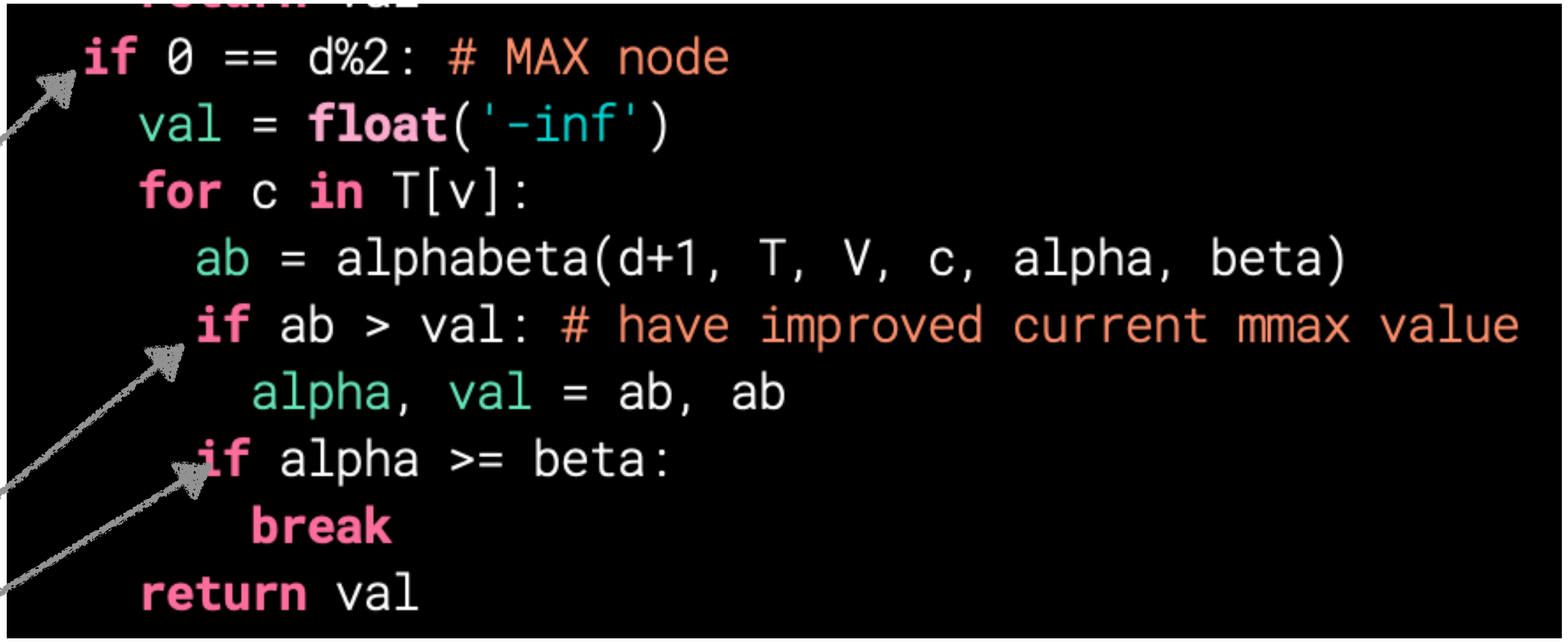
Implementation: abeta/alphabeta.py



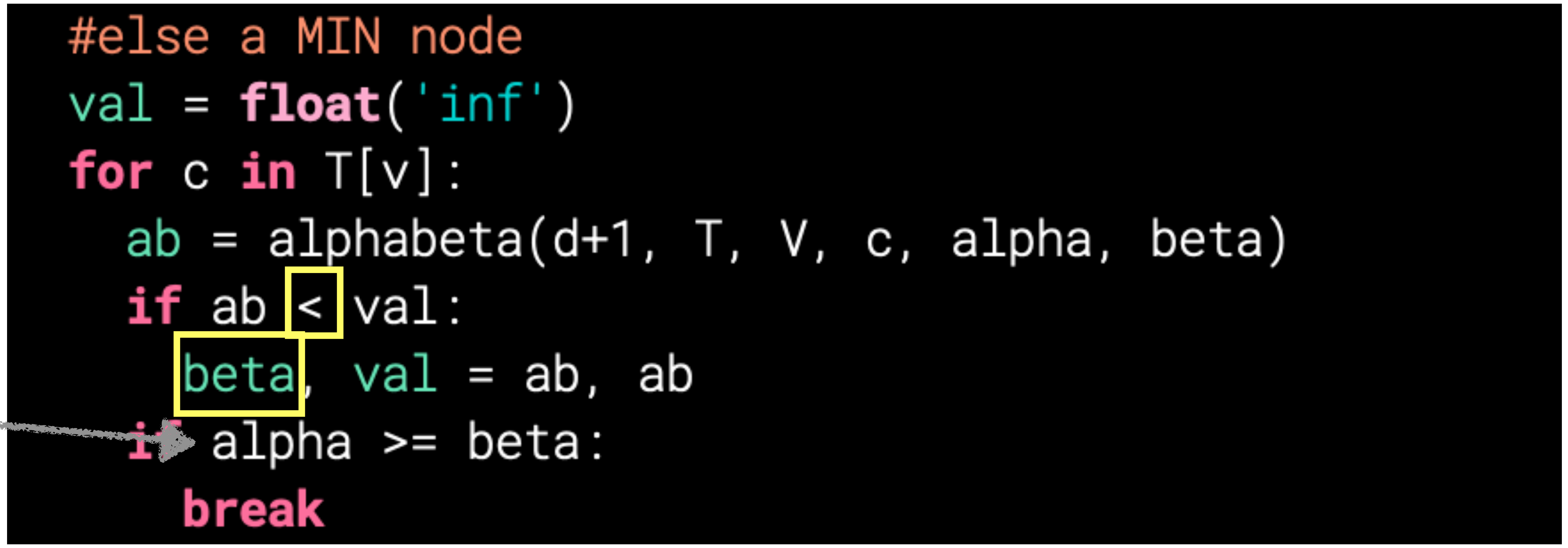
```
L,T,V,root = readtree()
alphabeta(0, T, V, root, float('-inf'), float('inf'))
```



```
def alphabeta(d, T, V, v, alpha, beta):
  if v in V: # V is the set of leaves
    val = V[v]
    return val
```



```
  if 0 == d%2: # MAX node
    val = float('-inf')
    for c in T[v]:
      ab = alphabeta(d+1, T, V, c, alpha, beta)
      if ab > val: # have improved current mmax value
        alpha, val = ab, ab
      if alpha >= beta:
        break
    return val
```

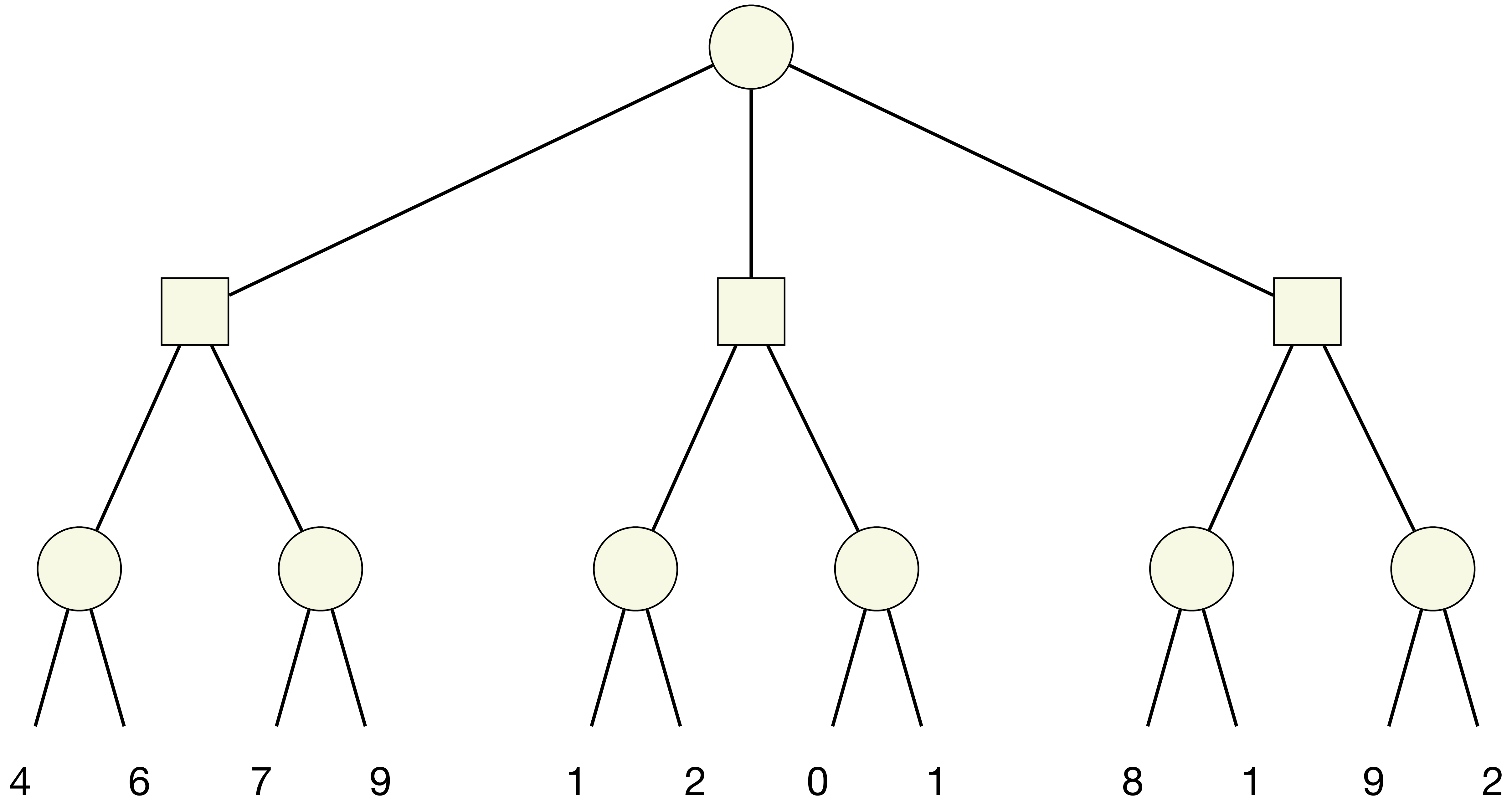


```
  #else a MIN node
  val = float('inf')
  for c in T[v]:
    ab = alphabeta(d+1, T, V, c, alpha, beta)
    if ab < val:
      beta, val = ab, ab
    if alpha >= beta:
      break
```

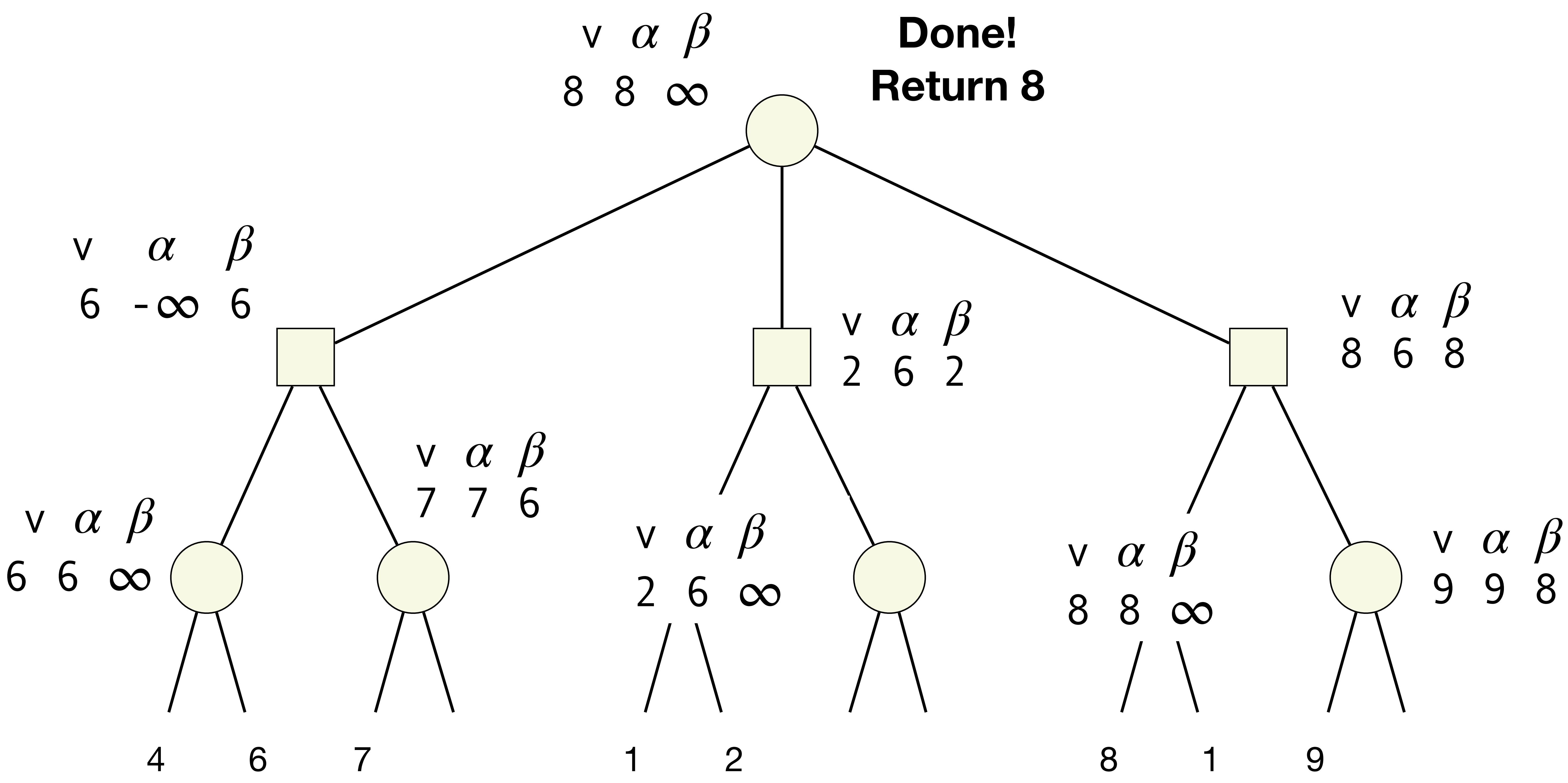
Question:

1. Why are we updating `alpha` if `ab > val` instead of `ab > alpha`?
2. How does this line tell us we're a MAX node?
3. Why `alpha >= beta` instead of `alpha > beta`?

Alpha-Beta Search Example #2



Alpha-Beta Search Example #2



Summary

- Minimax search examines every node in the search graph
- But when an ancestor **max** node has an option that is **higher** than the current subtree's value, we'll never reach this subtree (in optimal play)
- Similarly, when an ancestor **min** node has an option that has **lower** than the current subtree's value, then we'll never reach this subtree
- So, can stop exploring subtree once you prove that it has:
    - **lower value** than **ancestor max** node's best option, *or*
    - **higher value** than **ancestor min** node's best option
- **Alpha-beta search** checks before each recursive call whether subtree is optimally reachable
    - **Alpha: max value** option for any **ancestor max** node (possibly including self)
    - **Beta: min value** option for any **ancestor min** node (possibly including self)