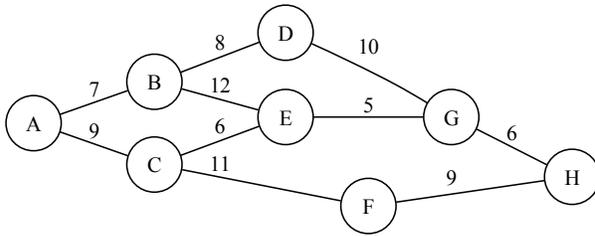


1. [5 points] Consider the following search graph. Edges are labelled with costs, and the table beneath gives the heuristic value for each node. The goal is to find a path from A to H. Recall that a node is *expanded* when it is removed from the fringe. List the order in which nodes are expanded by A*, and contents of the fringe after each expansion. When two nodes have identical priorities, expand the one that comes alphabetically first (i.e., break ties in alphabetical order).



| | | | | | | | | |
|--------|----|----|----|----|----|---|---|---|
| n | A | B | C | D | E | F | G | H |
| $h(n)$ | 20 | 17 | 15 | 12 | 10 | 9 | 6 | 0 |

| Node | Fringe |
|-----------------|------------------|
| Expand A | B:24, C:24 |
| Expand B | C:24, D:27, E:29 |
| Expand C | D:27, E:25, F:29 |
| Expand E | D:27, F:29, G:26 |
| Expand G | D:27, F:29, H:26 |
| Expand H (done) | D:27, F:29 |

2. [4 points] Consider the following 6×6 sliding tile position:

| | | | | | |
|----|----|----|----|----|----|
| | 3 | 2 | 1 | 5 | 4 |
| 6 | 7 | 8 | 9 | 10 | 11 |
| 12 | 13 | 14 | 15 | 16 | 17 |
| 18 | 19 | 20 | 21 | 22 | 23 |
| 24 | 25 | 26 | 27 | 28 | 29 |
| 30 | 31 | 32 | 33 | 35 | 34 |

- (a) How many *inversions* does this position have? **5 inversions:** (3, 2), (3, 1), (2, 1) are out of order, but 1, 2, 3 are correctly ordered with respect to all later numbers; (5, 4) are out of order, but correctly ordered with respect to all other numbers; (35, 34) are out of order, but correctly ordered with respect to all earlier numbers.
- (b) How many *misplaced tiles* does this position have? **33 misplaced tiles:** 35 and 5 are correctly placed; every other tile is misplaced.
- (c) What is the *taxicab distance* for this position?
 1 needs to travel 3 left; 2 needs to travel 1 left; 3 needs to travel 1 right. 4 needs to travel 2 left. 5 is correctly placed. 6,12,18,24,30 need to travel 1 up and 5 right. 7–11,13–17,19–23,25–29, 31–33 need to travel 1 left. 35 is correctly placed; 34 needs to travel 2 left.
 $3+1+1+2+5(6)+23(1)+2 = \mathbf{62}$.
- (d) Is this position *solvable*? Why or why not? **Yes; there are an even number of columns, and there are an odd number of inversions and the blank height is odd. Equivalently, (number of inversions + blank height) is even.**

3. [4 points] Suppose that running breadth-first search on a 8×3 sliding tile puzzle takes 0.5s in the worst case.

- (a) How many possible 8×3 positions are there? (You don't need to expand the expression to get an actual number)
 $24!$
- (b) What is the average degree (i.e., number of neighbours) of a 8×3 position?

| | | |
|---|---|---|
| 2 | 3 | 2 |
| 3 | 4 | 3 |
| 3 | 4 | 3 |
| 3 | 4 | 3 |
| 3 | 4 | 3 |
| 3 | 4 | 3 |
| 3 | 4 | 3 |
| 2 | 3 | 2 |

4/24 corners have 2 neighbours; 14/24 edges have 3 neighbours; 6/24 central cells have 4 neighbours.

$$\frac{4}{24}2 + \frac{14}{24}3 + \frac{6}{24}4 = \frac{74}{24} = \frac{37}{12}.$$

- (c) What is the average degree (i.e., number of neighbours) of a 4×6 position?

| | | | | | |
|---|---|---|---|---|---|
| 2 | 3 | 3 | 3 | 3 | 2 |
| 3 | 4 | 4 | 4 | 4 | 3 |
| 3 | 4 | 4 | 4 | 4 | 3 |
| 2 | 3 | 3 | 3 | 3 | 2 |

4/24 corners have 2 neighbours; 12/24 edges have 3 neighbours; 8/24 central cells have 4 neighbours.

$$\frac{4}{24}2 + \frac{12}{24}3 + \frac{8}{24}4 = \frac{76}{24} = \frac{19}{6}.$$

- (d) Give an estimate of how long it would take to run breadth-first search on an unsolvable 4×6 position. Justify your answer. $0.5s \times \frac{24!76}{24!74} = \frac{38}{74}s$

4. [2 points] Recall that `stile/15puzzle.py` has three different subgoal schedules:

- (i) `[[1], [2], [3,4], [5], [6], [7,8], [9,13], [10,14], [11,12,15]]`
- (ii) `[[1,2], [3,4], [5,6,7,8], [9,10,11,12,13,14,15]]`
- (iii) `[[1,2,3,4], [5,9,13], [6,7,8,10,11,12,14,15]]`

schedule (i) places tile {1} first, then tile {2}, etc. Schedule (ii) places tiles {1,2} first, then tiles {3,4}; Suppose that we run `stile/15puzzle.py` on a solvable position.

- (a) Which schedule is likely to produce the *shortest solution*? Why?
 Schedule (iii), because it has a small number of large subgoals.
- (b) Which schedule is likely to *search the fewest nodes*? Why?
 Schedule (i), because each subgoal is very small (and will therefore have a very small search space).