1. Consider the following 6×6 sliding tile position:

	3	2	1	4	5
6	7	8	9	10	11
12	13	14	15	16	17
18	19	20	21	22	23
24	25	26	27	28	29
30	31	32	33	34	35

   (a) How many *inversions* does this position have?

   (b) Is this position *solvable*? Why or why not?

   (c) What is the *taxicab distance* for this position?

   (d) How many *misplaced tiles* does this position have?

2. Consider the following 3×3 sliding tile position:

8	6	7
2	5	4
3		1

   (a) How many *inversions* does this position have?

   (b) Is this position *solvable*? Why or why not?

   (c) Give a non-vacuous (i.e., not 0) lower bound on the number of moves required to solve this position, and explain your reasoning.

3. Are the following *admissible* heuristics for the sliding tile puzzle? Why or why not?

   (a) 2 times taxicab: Returns double the taxicab distance.

   (b) $\frac{1}{2}$ times taxicab: Returns half the taxicab distance.

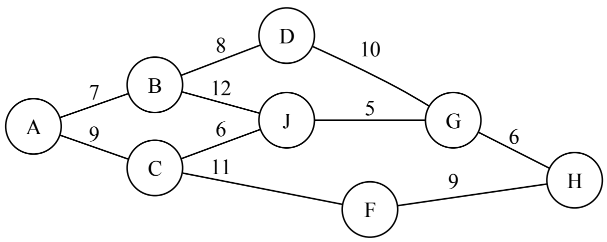
4. Which heuristic is more accurate: taxicab distance or number of misplacements? Explain why.

5. For a general graph, under what circumstances is A* guaranteed to return:

   (a) The *cheapest* solution?

   (b) The *shortest* solution?

6. Consider the following search graph. Edges are labelled with costs, and the table beneath gives the heuristic value for each node. The goal is to find a path from A to H. Recall that a node is *expanded* when it is removed from the fringe. List the order in which nodes are expanded by A*, and contents of the fringe after each expansion. When two nodes have identical priorities, expand the one that comes alphabetically first (i.e., break ties in alphabetical order).



n	A	B	C	D	J	F	G	H
$h(n)$	20	17	15	12	10	9	6	0

Node	**Fringe**
Expand A	B:24, C:24

7. For the same search graph, again breaking ties alphabetically:

   (a) What solution (i.e., path from A to H) will be found by *breadth-first search*? Explain why.

   (b) What solution (i.e., path from A to H) will be found by *depth-first search*? Explain why.

   (c) In what sense is *random search* guaranteed to find a path to H?

   (d) In what sense is random search *not guaranteed* to find a path to H?

8. Suppose that running breadth-first search on a 6×2 sliding tile puzzle takes 0.25s in the worst case (i.e., starting from an unsolvable position).

   (a) How many possible 6×2 positions are there? (You don't need to expand the expression to get an actual number)

   (b) What is the average degree (i.e., number of neighbours) of a 6×2 position?

   (c) What is the average degree (i.e., number of neighbours) of a 3×4 position?

   (d) Give an estimate of how long it would take to run breadth-first search on an unsolvable 3×4 position. Justify your answer.

9. Recall that `stile/15puzzle.py` has three different subgoal schedules:

   (i) `[[1,2,3,4],[5,9,13],[6,7,8,10,11,12,14,15]]`

   (ii) `[[1],[2],[3,4],[5],[6],[7,8],[9,13],[10,14],[11,12,15]]`

   (iii) `[[1,2],[3,4],[5,6,7,8],[9,10,11,12,13,14,15]]`

   Schedule (i) places tiles $\{1, 2, 3, 4\}$ first, then tiles $\{5, 9, 13\}$; schedule (ii) places tile $\{1\}$ first, then tile $\{2\}$, etc. Suppose that we run `stile/15puzzle.py` on a solvable position.

   (a) How many positions are in the search graph for the first subgoal of each schedule?

   (b) How many positions are in the search graph for the *second* subgoal of each schedule?

   (c) Which schedule is likely to produce the *shortest solution*? Why?

   (d) Which schedule is likely to *search the fewest nodes*? Why?

10. Suppose that `stile/15puzzle.py` is run on a solvable position, and successfully completes the first three subgoals of schedule (iii). Is the fourth subgoal guaranteed to be solvable on the resulting position? Why or why not?

11. Prove the following claim: For a 4×5 sliding tile puzzle (i.e., 4 rows, 5 columns), every move leaves the parity of the number of inversions unchanged.

12. Prove the following claim: There is no sequence of moves from a solvable sliding tile position to an unsolvable one.

13. Consider the following 3×3 sliding tile position:

4	3	2
1	5	6
7	8	

(a) What *arrangement* does this position correspond to?

(b) Give a sequence of moves that places the blank in the center square of the top row, without changing the arrangement.

(c) Give a sequence of moves that places 6 before 3 in the arrangement, while leaving the rest of the arrangement unchanged.

14. Here are two 3×7 sliding tile positions. Some of the numbers are hidden.

?	?	?	?	?	?	?
?	?	?	?	?	?	14
12	11	16	19	2	3	

?	?	?	?		4	15
7	22	11	24	18	?	?
?	?	?	?	?	?	?

(a) In the left position, what is the change in the number of inversions after the move **14 down**?

(b) In the right position, what is the change in the number of inversions after the move **18 up**?

(c) In the right position, what is the change in the number of inversions after the move **4 left**?