Computing Science (CMPUT) 455

Search, Knowledge, and Simulations

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455 Today - Lecture 5

Today's Topics:

- More on size of state space, effort of solving a game
- Sequential decision-making

Coursework and New Uploads

- Quiz 3
- Assignment 1
- Reading O'Neil, How algorithms rule our working lives
- Activities Lecture 5
- Python codes `count_dag.py`, `generate_tree.py`, `generate_tree_test.py`

Activities 5a and 5b

- Compute size of game trees
- When does a tree grow more quickly?
  - When increasing *b*?
  - When increasing *d*?
- Compare two cases

Size and Structure of State Space for Games

Review: State Space vs Playing and Solving a Game

- What is the complexity of solving, or playing well, in a game?
- Depends on many factors:
  - Branching factor
  - Depth
  - Existence of a simple strategy
  - Existence of a mathematical theory
  - Having master players, master games, books to learn from
  - Having good heuristics
  - ...

Review: State Space vs Playing and Solving a Game

Simple measures of complexity of state space

- Size: how many states?
- Structure: tree, DAG, or DCG?
- Branching factor b
- Depth d

Estimating Search Effort

- Assume we need to visit every state
  in order to solve a game
  - (Later we will see we can do much better)
- How long does it take?
- Main factors:
  - Speed of program
  - Size of state space
- Let's look at 7×7 Go and `Go1`

7×7 Go - Example

- 7×7 Go, start on empty board
- Assume we can process 1000 states/second
- Assume simplest tree model, $b = 49$
- What depth d can we reach in which time?
- Can we explore the whole state space?

Brute Force Search for 7 $\times$ 7 Go

Table: Each row shows the estimated *additional effort* to search one level deeper

Depth	New states	Added search time
0	1	1ms
1	49	50ms
2	49^2	2.4s
3	49^3	2 min
4	49^4	1.6 hrs
5	49^5	3.2 days
6	49^6	160 days
7	49^7	21.5 years
...	...	...

Fighting Exponential Growth

- We cannot even search 7 moves deep with `Go1`
- To solve the game we need to see to the end
- This can be over 30 moves deep even for this small board
- Time limits in practice
  - 5 sec - 5 min per move in a tournament game
  - Maybe a few months to solve a game
- How can we succeed?
  - Increase speed of program (Lecture 6)
  - Decrease branching factor *b* (now)
  - Decrease depth *d* (later)

Decrease Branching Factor *b*

- Branching factor = growth of number of states per level
- How to decrease?
  - Reduce number of moves (but how?)
  - Use DAG instead of tree
  - Better search algorithms (e.g. alphabeta search)

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- Question: How would using a DAG help decrease the branching factor?

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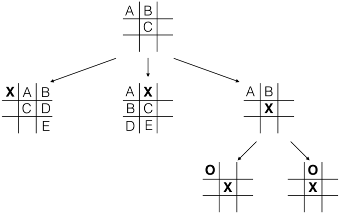
- Question: How would using a DAG help decrease the branching factor?
- First try: take symmetry into account

Example - Use Symmetry in TicTacToe

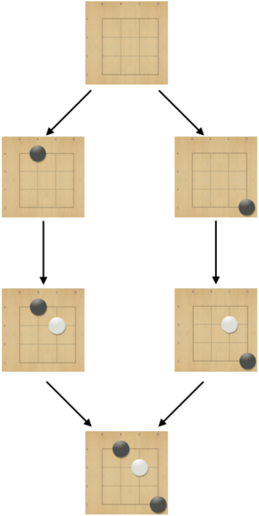


- At root: Only 3 of 9 total moves are different
  - Corner (A), Edge (B), Center (C)
- All 6 other moves lead to a symmetric position, same result as A or B
- Symmetries at tree level 1:
  - After corner or edge move: 5 distinct cases
  - After center move: 2 distinct cases
- Limitation: most symmetries broken after few moves

Symmetry

- Typical example to reduce state space by symmetry
- Good reduction at depth 1 or 2
- Then *symmetry breaks*
- Almost no reduction deeper in the tree
- Reduction of whole state space is limited to some constant factor
    - Less than 8 in Go

DAG (Directed Acyclic Graph)



- Idea: single node for all *equivalent* states
- Different paths to same node
- Can lead to huge reduction in state space
- Why?
  - The whole *subtree* below is no longer duplicated
  - Can happen throughout the whole game

Tree vs DAG

- Tree model
  - Each action leads to a new node
- DAG model
  - All *equivalent* states represented by a single node
- Reduction in size of state space
  - Can be many orders of magnitude
  - Examples: Activities 5c - 5e
- Advantages of DAG model:
- Avoid redundant computations
  - No copied subtrees or sub-DAGs
- Share results of analysis - compute once, re-use often

Limitations of DAG Model

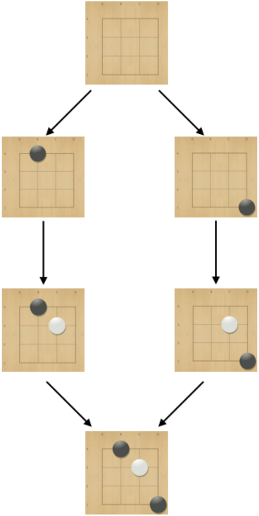
- Main problems:
  - Need memory to store and recognize equivalent states
  - Some algorithms designed only for trees, not for DAGs
- Example: propagating information up towards root
  - Only one path up in tree - efficient
  - Many paths in DAG - many ancestors

More Limitations of DAG Model

Limitation: states with different *history*

- Cannot always merge into one node, not always equivalent
- Example: simple ko - is capture allowed?
- Board looks the same but moves are different
- Can you still do something? Yes. One of Martin's students wrote a whole PhD thesis on such questions (Kishimoto 2005)

Counting States in a DAG



- Simplified GoMoku example, also counts illegal states
- Depth 0: 0 black, 0 white stones
- Depth 1: 1 black, 0 white stones
- Depth 2: 1 black, 1 white stones
- Depth 3: 2 black, 1 white stones
- Depth d: $\lceil d/2 \rceil$ black stones and $\lfloor d/2 \rfloor$ white stones
- How many ways to put that many stones on a board with 49 points?

Counting States in a DAG (continued)

- Example:
- Board with 49 squares
- How many different ways to place 5 black stones?
- Answer: $\binom{49}{5} = 1,906,884$
- Need to review the math background? See e.g. `https://en.wikipedia.org/wiki/Combination`

Counting States in a DAG (continued)

- How many different ways to place 5 black stones and 3 white stones?
- Answer: $\binom{49}{5} \times \binom{44}{3} = 1,906,884 \times 13,244 \approx 25.2$ billion
- Why?
  - $\binom{49}{5}$ ways to place 5 black stones
  - $49 - 5 = 44$ empty points remaining
  - $\binom{44}{3}$ ways to place the 3 white stones there
  - Each different choice for either black or white leads to a different position, so multiply

Activity 5d: Count States in Tic-Tac-Toe DAG

- TicTacToe board has 9 squares
- Ignore symmetry, early wins for now
- At level d: $\lceil d/2 \rceil$ X's and $\lfloor d/2 \rfloor$ O's
- Compute number of positions with 0, 1, 2, 3, $\ldots$, 9 stones

0 stones	0 X, 0 O	1 position
1 stone	1 X, 0 O	9 positions
2 stones	1 X, 1 O	? positions
3 stones	2 X, 1 O	? positions
4 stones	2 X, 2 O	? positions
…		? positions
9 stones	5 X, 4 O	? positions

Hint: Python code in `count_dag.py` is useful...

b^d Model and DAG

- Counted the number of states in a DAG at each depth d
- What about the branching factor?
- No longer a constant b for whole DAG
- Different *effective branching factors* b_d depending for each depth (or level) d
- Can compute b_d at depth d as:

$$b_d = \frac{\textit{\#nodes-at-depth } d + 1}{\textit{\#nodes-at-depth } d}$$

- Activity 5e: compute the effective branching factor for the TicTacToe DAG

Computing Size of State Space in DAG from Branching Factors

- Given branching factors, how many nodes in a DAG?
- 1 root node at depth 0
- b_0 children of root $\longrightarrow$ b_0 total nodes at depth 1
- Each child has b_1 new children
  $\longrightarrow$ total $b_0 \times b_1$ nodes at depth 2
- Depth n:
  $b_0 \times b_1 \times ... \times b_{n-1}$ nodes

Computing Size from Branching Factors (continued)

- Total nodes up to depth d
  1
  $+ b_0$
  $+ b_0 \times b_1$
  $+ \ldots$
  $+ b_0 \times b_1 \times \ldots \times b_{d-1}$
- In general:
  no nice closed-form solution for this sum
- In practice:
  estimate branching factors b_i
  - Use search or sampling
  - Difficult or impossible to compute them exactly for large games

b^d Model vs Reality: Some Case Studies (1)

- How realistic is the b^d model for size of state space?
- We'll look at some popular games
  - Go, TicTacToe:
  - Roughly, $b_n \approx b_0 - n$
  - Why? One less empty square with each move
  - One less possibility for next move
- Not exact:
  - Ignores illegal move rules
  - Ignores games that end earlier
- Setting $b_n = b_0 - n$ gives $b_0!$ leaf nodes
  - Earlier TicTacToe example: $9 \times 8 \times \ldots 1$

b^d Model vs New estimate: 7×7 Go

- 7×7 Go estimate from Lecture 4:
- 25 moves on average during a game, game length about 30 moves
- Rough b^d estimate $25^{30} \approx 10^{42}$
- New model:
- $b_0 = 49$, and $b_n \approx b_0 - n$
- Stop game at $n = 30$
- $49 \times 48 \times ... \times (49 - 30) = 49!/18! \approx 10^{47}$
- Still ignores captures, ko, different game lengths,...

b^d Model vs Reality: Checkers

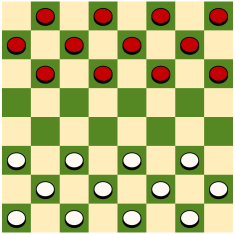


Image source:

`https://en.wikipedia.org`

- Checkers: complicated b
- Beginning: many pieces blocked
- Pieces unblocked: b increases
- Forced captures: $b = 1$
- When pieces get captured, b decreases again
- When checkers become kings, b strongly increases
- Estimated average over typical game: $b \approx 2.8$
- Length of game d varies wildly

b^d Model vs Reality: Chess

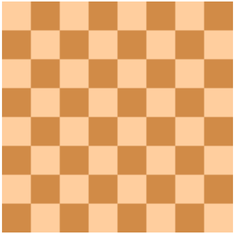


Image source:

`https://en.wikipedia.org`

- Chess: also complicated
- Pieces such as queens can have many moves, but may be blocked
- King in check:
  often only few legal moves
- When pieces get captured,
  b decreases
- Estimated average over typical game: $b \approx 35$
- Length of game d varies wildly

b^d Model vs Reality: Shogi

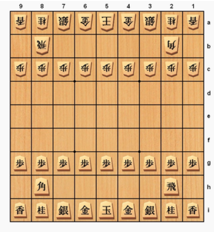


Image source:

`https://en.wikipedia.org`

- Shogi, Japanese chess
- Similar to chess, plus:
- Captured opponent pieces can be *reused* for yourself in a future move
- With captures, *b increases*
- Estimated average over typical game: $b \approx 92$
- *b* can be several hundred in endgame with many captured pieces available for “dropping” back on board

Complexity of Popular Games

- Big table in
  `https://en.wikipedia.org/wiki/Game_complexity`
- Different measures of complexity
- Complexity also depends strongly on size of board
- In Go, the theoretical complexity is much higher
  - Main reason: capture, play again on same point
  - Example: `Go0` player fills eyes, games last VERY long
  - Game ends only if **all** moves cause full-board repetition

Example: Solving 2×2 Go, $1 \times n$ Go

- 2×2 Example from John Tromp, `https://tromp.github.io/java/go/twoxtwo.html`
- Naive brute force minimax search: trillions of nodes
- Alphabeta with bad move ordering:
  - 19,397,529 nodes, max. depth 58
- Alphabeta with good move ordering:
  - 1,446 nodes, max. depth 22
- Solving $1 \times n$ Go: *Exploring Positional Linear Go* recent paper by Noah Weninger (UofA undergrad!) and Ryan Hayward (UofA prof) - see resources

Sequential Decision-Making

Sequential Decision-Making

Topics:

- From decision making to sequential decision making
- Notation for action sequences
- View game tree as tree of move sequences

From State Space to Decision-Making

- We studied state spaces in some detail
- Now, how do we find good actions (moves) in a state?
- In general, looking at the current state is not enough
- We need to look ahead to future states in order to make a good decision now
- We need to consider sequences of actions, until we reach a terminal state
- In games, each sequence is one possible way of playing the game

Making Complex Decisions

- How to make good decisions?
- Consider many alternatives
- Consider short-term and long-term consequences
- Evaluate different options and choose the best-looking one
- Understanding and comparing sequences of actions is the main step in making such decisions

Making Sequential Decisions

Very general model:

Loop:

- Get current state of world
- Analyze it
- Select an action
- Observe the world's response
- If not done: go back to start of loop

Practically important question:

- Can we do this in a *simulation model* as opposed to the real world?

Single Agent Example: Path Planning

Image source: `Googlemaps`

- Task: start from here, visit Google headquarters
- First decision:
  fly, drive, take the bus, or walk?
- If drive or walk: each street corner is a decision point
- Need a long sequence of decisions to arrive at destination
- Is it *optimal*? Is it *good enough*?
- Tradeoffs: Speed vs cost vs scenery vs construction sites ...

Formal Framework

- Sequence of states and actions
- Start state s_0
- Action a_i leads to next state, s_{i+1}
- Keep going until reach a terminal state s_n
- Sequence $(s_0, a_0, s_1, a_1, ...s_n)$
- Sometimes we only write the actions $(a_0, a_1, ...a_n)$
  - Example: games where states are determined from game rules and actions

Formal Framework with Rewards

- Formal framework can also include rewards (or costs)
- Simple case (most games we consider):
  single reward r at end
- General case: reward r_i after each action a_i
  - Write rewards as part of sequence:
  - $(s_0, a_0, r_1, s_1, a_1, r_2, ..., r_n, s_n)$

Partial Sequences

- Full sequence goes all the way to terminal state
- *Partial sequence* can stop after any number of actions
- Two full sequences always share a *common prefix*
- In worst case, it might only contain the start state
  - Example - Go game
  - $(s_0, \text{Black B3}, s_1, \text{White A2}, s_{2a}, \text{Black D4})$
  - $(s_0, \text{Black B3}, s_1, \text{White A4}, s_{2b}, \text{Black D4})$
  - Common prefix $(s_0, \text{Black B3}, s_1)$

Re-Interpreting the Tree and DAG Models

- Our model so far:
- State space as a graph
- Nodes are states, edges are actions
- Tree and DAG are special cases of graphs
- New view:
  we can view the same trees and DAGs
  as a way to organize all action sequences

Organizing Sequences in Trees

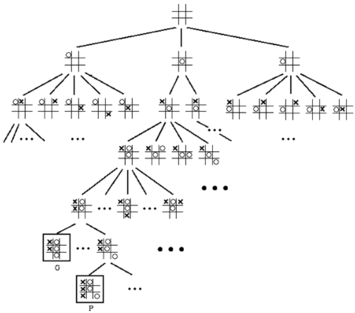
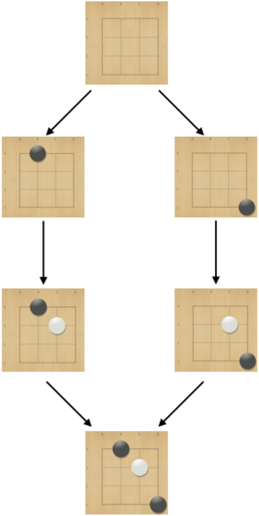


Image source: `http://web.emn.fr`

- Consider the (huge) set of all possible state-action sequences
- Organize them such that:
- Any two sequences share their *longest common prefix*
- Branch as soon as they differ
- Result: we get exactly the tree representation of the state space

Organizing Sequences in a DAG



- Similarly, we can relate sequences to the DAG model
- Start with sequences-as-tree model
- Then, merge two different sequences when they both reach equivalent states
- Result: we get exactly the DAG representation

Summary

- Looked at more details and examples of state spaces
- Estimating size of state space in DAG model
- Theory vs reality: state space of some popular games
- Sequential decision-making model
- Relation between tree and DAG models, sequences of decisions

Preview - What's Next?

- Lecture 6: profiling and optimization
- Topics for next few weeks:
  - Do we need to look at *all* possible sequences to make a decision?
  - How do we use the tree or DAG structure?
  - Algorithms for decision-making in games on tree and DAG structures
  - How do we use heuristics?
  - Simulation using random sequences of actions