Computing Science (CMPUT) 455

Search, Knowledge, and Simulations

James Wright

Department of Computing Science
University of Alberta
james.wright@ualberta.ca

Fall 2021

455 Today - Lecture 4

Topics:

- Formalizing decision making, search and games
- Basic concepts - states, actions, state space
- Examples

Coursework

- Continue working on assignment 1 (due on Monday)
- Designated submitter: email your team information to a TA
  - See assignments page for format
- Activities for Lecture 4

Lecture 4: Formalizing Decision-Making

Lecture Topics

- Intro to computer decision-making
- Search, state spaces, state space of a game
- Terminology for state space search
- Types and models of state spaces
- Game trees, b^d model

Decision-making Exercise - Use Computers?

- Could a computer do the same decision-making processes as you?
- What would a computer need to make similar decisions?
  - Knowledge about your problem?
  - Knowledge about the world, “common sense”?
  - Logical reasoning?
  - Optimization?
  - Input? Sensors? Memory? Processing power?

Do you Want Computers to Make Decisions?

- Would you want a computer to make decisions **for you**?
- When would you trust it?
- How about computers supporting human decision-making?
- Who is in control?
  - Machine?
  - Programmer?
  - Employer?
  - Government?
  - Nobody?

Reasoning - Now vs Future

- Some decision-making considers only the present
- Reflexes
- Intuitive Decisions
- Tasks where looking ahead is not needed

Reasoning - Now vs Future

- Some decision-making considers only the present
- Reflexes
- Intuitive Decisions
- Tasks where looking ahead is not needed
- Example: image recognition
  - All relevant knowledge is available now
  - Neural net has learned knowledge from many (past) examples

Reasoning - Now vs Future

- Much reasoning involves thinking about the future,
  in order to make decisions now

Reasoning - Now vs Future

- Much reasoning involves thinking about the future, in order to make decisions now
- Example: should I take an umbrella to work?
  - It is not not raining now
  - The forecast predicts rain later today

Reasoning - Now vs Future

- Much reasoning involves thinking about the future, in order to make decisions now
- Example: should I take an umbrella to work?
  - It is not not raining now
  - The forecast predicts rain later today
- Chess example: should I capture this queen?
  - It looks good now
  - It gets me checkmated 8 moves later

Decision Making as Search Problem

- Decision making in practice has many complications
- Often, we do not know:
  - The possible actions
  - The goals of others
  - How to evaluate different outcomes
  - How to compute with limited resources
  - ...

It is difficult to make predictions,
especially about the future.

`https://quoteinvestigator.com/2013/10/20/no-predict/`

Decision Making as Search Problem

- Decision making in practice has many complications
- Often, we do not know:
  - The possible actions
  - The goals of others
  - How to evaluate different outcomes
  - How to compute with limited resources
  - ...

  It is difficult to make predictions,
  especially about the future.

  `https://quoteinvestigator.com/2013/10/20/no-predict/`

- Where do we start? Simplify. A lot.

A Simple Setting for Decision-Making

Long list of assumptions:

- The *state* of the world is completely known at each time
- There are *terminal states* where we can evaluate the result precisely, e.g.
  - A number: value, score, reward, utility,...
  - One of a set of possible outcomes, e.g. {win, loss, draw}
- The possible *actions* (or moves)
  in each state are known
- An action changes the state to a new state in a known, deterministic way
- No chance element -
  no dice rolls, cards drawn, other random events

A Simple Setting for Decision-Making

Long list of assumptions:

- The *state* of the world is completely known at each time
- There are *terminal states* where we can evaluate the result precisely, e.g.
  - A number: value, score, reward, utility,...
  - One of a set of possible outcomes, e.g. {win, loss, draw}
- The possible *actions* (or moves)
  in each state are known
- An action changes the state to a new state in a known, deterministic way
- No chance element -
  no dice rolls, cards drawn, other random events

Question: Do games fit this simple setting?

Do Games fit Our Simple Setting?

- Single-player (puzzles): often, yes
  - Rubik's cube, solitaire, rushhour, sudoku, crossword puzzles, 15-puzzle
- **Two-player games**: our main topic - see separate slide
- Multi-player games: mixed
  - Yes: Chinese checkers, ...
  - No: most card games,
    all dice games,
    most family board games, ...

Do Games fit Our Simple Setting?

- Online and computer games: mostly, no
  - Simultaneous moves, often in real time
  - Map only partially known
  - Complex physics simulations
  - Many other complications...

“Classical” Two Player Games

- Our simple setting, plus:
- Two players, often called Black and White
- Players move alternately: I play, you play, I play,...
- A move instantly changes the state (no duration, no slow transitions)
- Simplest, most frequent case is *zero-sum*:
  my win is opponent’s loss
  - Opposite: cooperative games
- Examples: chess, checkers, Go, Tic-Tac-Toe, ...

Why Study Decision-Making using "Classical" Games?

- Simple, controlled environment
- Still hard to solve or play well
- Interesting for many people
- Games and results are easy to understand
- Playing games well requires good decision-making skills
- We can study the *core problems* of decision-making without being distracted by too many complications

Formalizing State Space Search

Terminology

The next few slides introduce the terms
we use to talk about state space search in general,
and specifically, games.

- game state, state
- state space, game graph, game tree
- board state, position
- move, action
- move sequence, history, game record
- score, value, evaluation, result

State, Game State

- Complete description of the current situation
- In games: board position or cards etc,
  toPlay (whose turn it is)
- State, plus rules of game, allow us to determine
  actions (moves)
- Often includes (parts of) history:
  - Sequence of moves from start of game to current position

State, Game State

- Complete description of the current situation
- In games: board position or cards etc,
  toPlay (whose turn it is)
- State, plus rules of game, allow us to determine
  actions (moves)
- Often includes (parts of) history:
  - Sequence of moves from start of game to current position
  - Question: Is it necessary to have additional history information in Go?

Example in `Go1`

- How is game state represented in `Go1`?
- In `board.py`
- Class `GoBoard`
- Contains 1D array `board`
  - Each array entry contains the color of one point
- Contains field `ko_recapture` to implement simple ko rule
- `current_player` (`BLACK` or `WHITE`)
- Other fields: `name`, `version`,...

Action, Move

- Leads from one state to another
- Move in games may include `toPlay`:
  color of the player who plays the move
- *Alternating* play:
  - `current_player` changes after each move
  - We do not need to specify color with the move (e.g. can just store it in the state)
- Move sequence, history, game record:
  all moves played in a game
- In `Go1`: move represented by index of point in array, and color
  - Used e.g. in `play_move(point, color)`

State vs History

- When is history needed?
  - Depends on rules, structure of search space
- Example: Ko (repetition), legal moves in Go depend on previous move history
- Example: TicTacToe does *not* need history
- Compare with *Markov Decision Problems* (MDP) in single agent search, *Markov property*:
  - History is irrelevant in MDP
  - Current state contains all relevant information

History-only State?

- How about having *only* history, no other information in state?

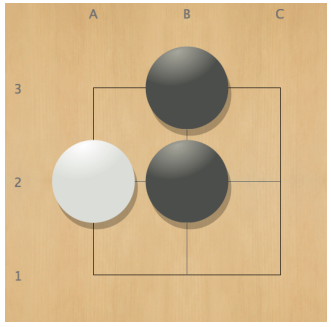
History-only State?

- How about having *only* history, no other information in state?
- Yes, that works in principle
  - Rules plus complete history determine the state exactly
  - Examples:
    - Game records with list of moves
    - Sequence of GTP play commands
- It may be inefficient if we always have to replay all actions from the beginning in order to find the current state
- Some forms of machine learning work with this representation

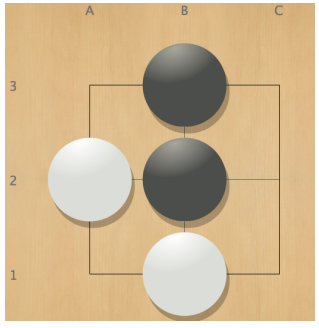
State Space

- A *state space* is:
  - A graph with all the possible states of a problem
  - Edges in graph show how states are connected by actions
- State space represented as directed graph $G = (V, E)$:
- Nodes in V: game states
- Directed edges in E: moves
  - Edge $e = (s_1, s_2)$ contains:
  - State s_1 before move
  - State s_2 after move

State Space Representation - `Go1` Example



`board1`



`board2`

- State before move:
  `(board1, to_play = WHITE, ko_recapture = None, ...)`
- Play move: White B1
- State after move:
  `(board2, to_play = BLACK, ko_recapture = None, ...)`

Terminal State

- A *terminal state* has no possible moves (actions)
- No outgoing edges in graph
- The rules of a game decide:
  - When is the game over?
    (did we reach a terminal state?)
  - What is the outcome in a terminal state?

Terminal State in Go

- Game can end in one of two ways
  - A player resigns
  - Both players pass in turn
- Most Go players do not keep playing until there are no legal moves
  - Some moves are bad anyway and should not be played (see `Go0` vs `Go1`)
- Stop playing when the ownership of each point is clear to both players, then count the score
- Example: `Go1.py` stops playing when there are only single point eyes left

Terminal States and Rewards

- Later, in reinforcement learning, we will talk a lot about *rewards* (or costs: negative rewards)
- In many games, the only reward is at the end, in the terminal state
- Example: +1 if you win, -1 if you lose
- A few games have other, earlier rewards/costs
  - Example: blinds and bidding in poker

Types of State Spaces

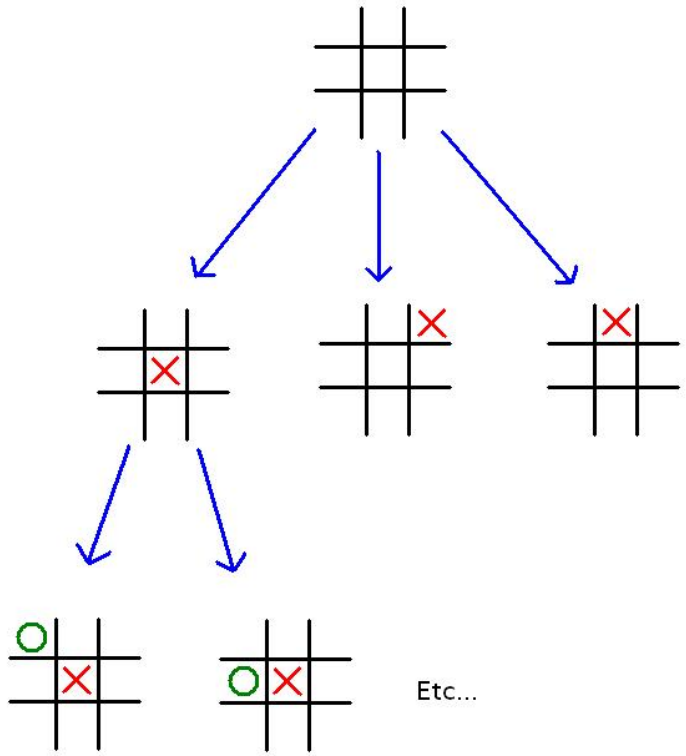
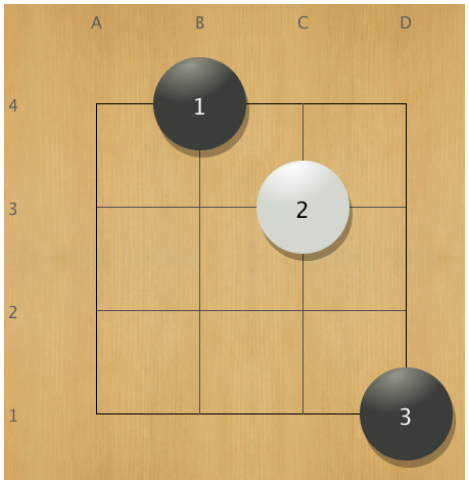


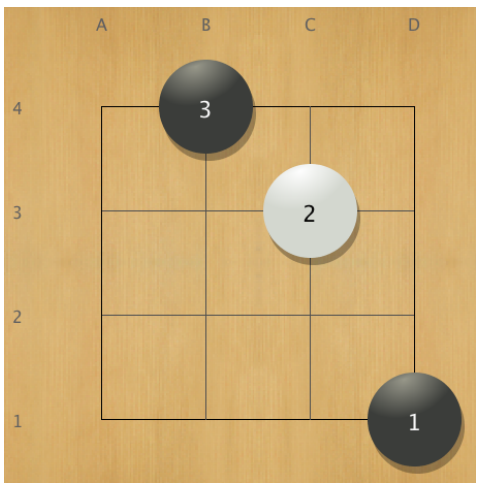
Image source:

`sciencefair.math.iit.edu`

- Assume *root* at the top is current state
  1. Tree
  2. DAG (directed acyclic graph)
  3. DCG (directed cyclic graph)
     - Tree is easiest for search, DCG hardest
- *Game graph, game tree* are other terms for state space of games

Examples of Types of State Spaces

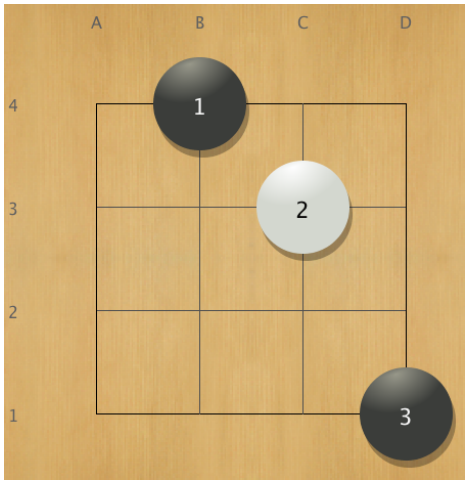


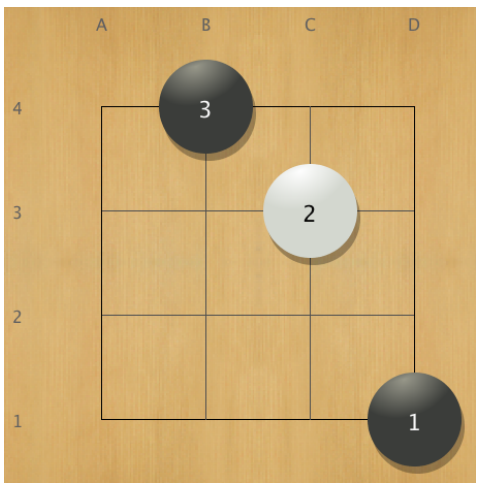


Same Go state reached with different *move order*

- Tic-Tac-Toe: DAG
  - Different move order can lead to same result
- Go without repetition rules: DCG (cycles exist, e.g. simple ko)
- Go with simple ko rules: still DCG (longer cycles still exist)
- Go with full repetition rules: DAG (details later)

Examples of Types of State Spaces

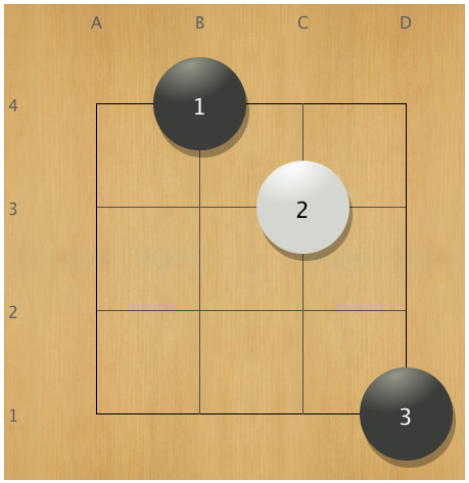


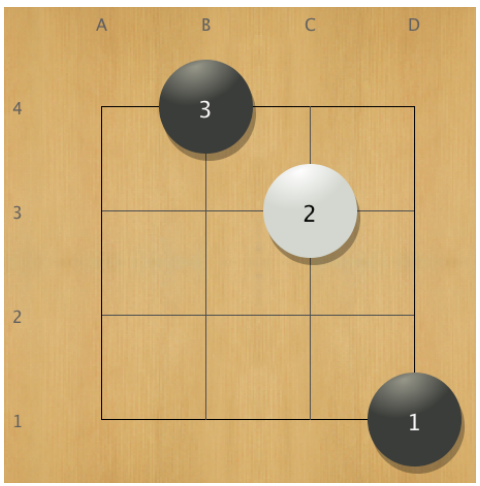


Same Go state reached with different *move order*

- Tic-Tac-Toe: DAG
  - Different move order can lead to same result
- Go without repetition rules: DCG (cycles exist, e.g. simple ko)
- Go with simple ko rules: still DCG (longer cycles still exist)
- Go with full repetition rules: DAG (details later)
- Question: What types of state space for the following games?
  - Chess?

Examples of Types of State Spaces

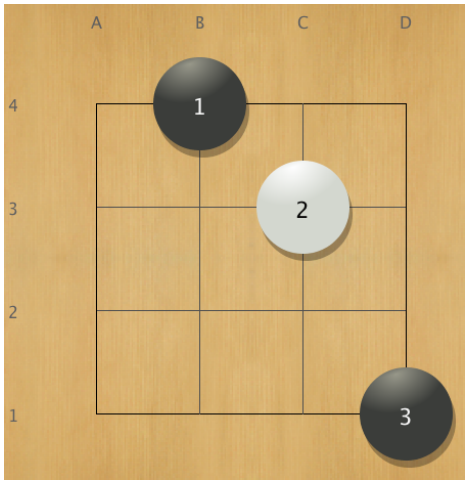


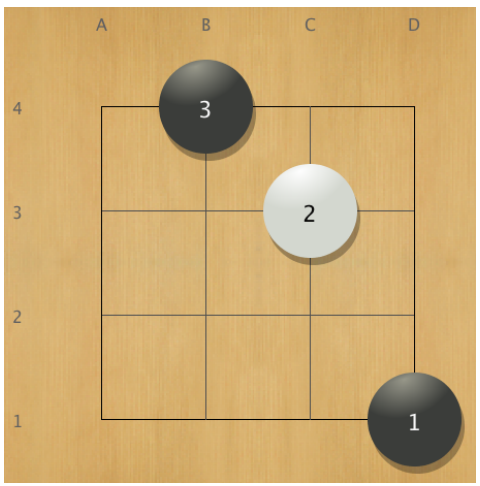


Same Go state reached with different *move order*

- Tic-Tac-Toe: DAG
  - Different move order can lead to same result
- Go without repetition rules: DCG (cycles exist, e.g. simple ko)
- Go with simple ko rules: still DCG (longer cycles still exist)
- Go with full repetition rules: DAG (details later)
- Question: What types of state space for the following games?
  - Chess?
  - Checkers?

Examples of Types of State Spaces





Same Go state reached with different *move order*

- Tic-Tac-Toe: DAG
  - Different move order can lead to same result
- Go without repetition rules: DCG (cycles exist, e.g. simple ko)
- Go with simple ko rules: still DCG (longer cycles still exist)
- Go with full repetition rules: DAG (details later)
- Question: What types of state space for the following games?
  - Chess?
  - Checkers?
  - GoMoku?

Complexity of a State Space

- Some measures of game complexity:
  - Size of state space
  - Branching factor (number of actions in state)
  - More on these later
- Difficulty of game can depend on many other things
  - Is there a simple strategy?
  - A mathematical theory?
  - Many master games to learn from?
  - Good heuristics?
- Difficulty for humans $\neq$ difficulty for computers

Game Result

- What is the result when a game is over?
- Simplest: win/loss
  - Go with non-integer komi, Nim, ...
- Many games: win/loss/draw
  - Chess, checkers, tic-tac-toe, Go with integer komi, GoMoku, ...
- Point-scoring games: size of win matters
  - Score, value, evaluation, result: terms with similar meaning

Types of Game Representation

- Board representation
- Move history - move list
- Formats:
  - Internal representation
  - File storage: move list + annotations, e.g. sgf file format
    - `www.red-bean.com/sgf/`
    - `en.wikipedia.org/wiki/Smart_Game_Format`
  - Inter-program communication: GTP - Go Text Protocol
    - `www.lysator.liu.se/~gunnar/gtp/`
    - Used to connect `GoO` etc to `GoGui` and other tools

Review - Board Representation

- Many games are grid-based
- 2D array
- 1D array (often faster, standard)
- Bitmaps (sometimes fastest, depends on use case)
- Specialized, e.g. piece list if large board, few pieces.

Summary - Concepts so Far

- State space search - formal model for decision-making
- Basic general concepts:
  states, actions, state space
- In games:
  game position, game state, move, game tree/game graph
- Terminal states, result, reward
- Representation of board and moves, internal and external
  (file storage, text communication)

Models of State Spaces

Models of State Spaces

- How to choose a state space for a decision-making problem?
- More on Tree, DAG, DCG
- Estimating the size of state space
- Reachable vs unreachable
- Symmetries, equivalent states

How to Choose States and State Spaces

- State needs all the *relevant information*
- Make it as simple as possible ...
- ... but not simpler

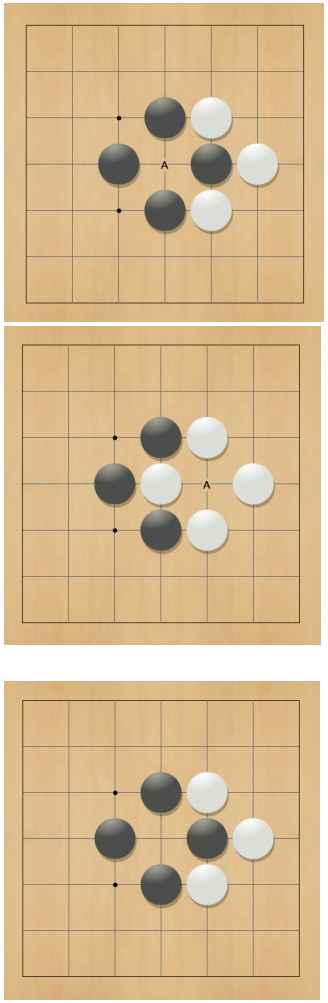
Example 1: Simplifying Rewards in States

- Example: a zero-sum game, both players collect rewards during the game
- Most direct representation:
  list of rewards at each move for each player
- Example of state:
  `([0, 10, 0, 0, 10], [20, 0, -5, 0, 0], rest-of-state)`
- Better: just keep sum of awards so far:
  `(20, 15, rest-of-state)`
- Even better: only keep *difference* of rewards
  `(5, rest-of-state)`

Example 1 Discussion

- Why is this representation better?
- All states with same reward difference are *equivalent*
  - The players do not care which of them they are in
- Great reduction in number of states
  - If one of the equivalent states is solved, then *all* are solved
  - Idea here: compress history into a single number, add that to state

Example 2: Simplifying State in Go

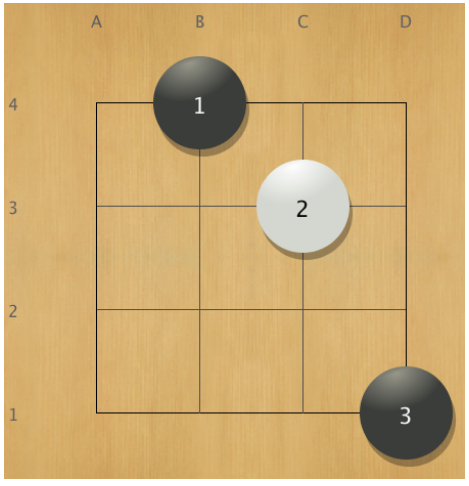


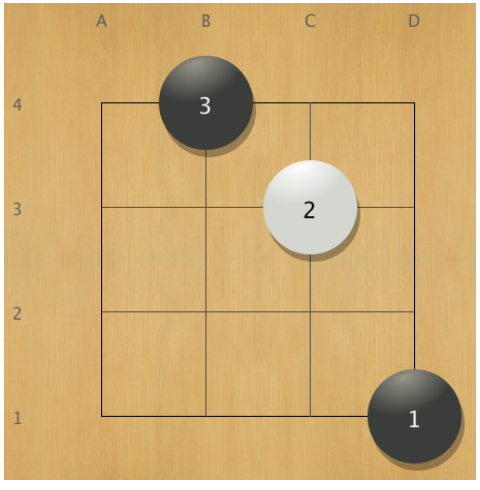
- Consider the *simple ko* rule in Go
- Need to check if the position is the same as two moves ago
- Simple way: store the positions, compare each point on board
- Better way: work out the conditions when the simple ko rule applies
- We need to only store a single point.
  - See `ko_recapture` in `Go1`
- Idea here: compress history into information of a single relevant point, add that to state

More on State Spaces - Tree, DAG, DCG

- What are the differences between tree, DAG, DCG?
- If we have a choice, which one to choose?
- We often have a choice!

State Space - Tree





- Simplest state space model is tree
- Every action leads to new state
- Only one path from root to a node
- We simply ignore it, if different paths lead to the same situation
- Each copy is a separate state
- Example: Two sequences of Go moves
- 1. Black B4, 2. White C3, 3. Black D1
- 1. Black D1, 2. White C3, 3. Black B4
- Different sequences, different states in tree model. Duplication!

Pros and Cons of Tree

Advantages

- Simplest model
- Single path to each node
- No dependencies

Disadvantages

- Duplication, no re-use of information
- State space can be much larger than needed
- Search can become very inefficient, searches many copies of equivalent sub-trees

Size of State Space - Simple Model

- How many states?
- Model:
- Assume a *constant branching factor b*
- Each *interior* node in tree has *b* children
- Assume a *uniform depth d*
- Each path from root is *d* actions long

State Space - Simple Model

- How many nodes?
- 1 root node, 1 = b^0 total nodes at depth 0
- b children of root, b^1 total nodes at depth 1
- Each child has b new children, total b^2 at depth 2
- ...
- Last level b^n nodes at depth n
- Total nodes $1 + b + ...b^n = (b^{n+1} - 1)/(b - 1)$
- For large b, this is close to b^n - last level dominates

(Bad) Example - Tic Tac Toe

- 9 possible moves, $b = 9$
- depth at most 9, $d = 9$
- So about $9^9 \approx 387$ million nodes?

(Bad) Example - Tic Tac Toe

- 9 possible moves, $b = 9$
- depth at most 9, $d = 9$
- So about $9^9 \approx 387$ million nodes?
- Question: Is this an accurate estimate? Why or why not?

A Better Model?

- Games such as TicTacToe only add stones, never remove
- n choices for first move
- $n - 1$ for second move, ...
- Total $n \times (n-1) \times ...1 = n!$ possible games
- TicTacToe: $9! = 362,880$

A Better Model?

- Games such as TicTacToe only add stones, never remove
- n choices for first move
- $n - 1$ for second move, ...
- Total $n \times (n-1) \times ...1 = n!$ possible games
- TicTacToe: $9! = 362,880$
- Question: Is *this* an accurate estimate? Why or why not?

(Very rough) Example - 7×7 Go

- 49 moves at start, maybe 25 on average during a game
- Length of game - about 30 moves?
- Rough estimate $25^{30} \approx 10^{42}$
- Compare with $49! \approx 10^{63}$

Summary - Models of State Spaces

- State space search - formal model for decision-making
- Basic general concepts: states, actions, state space
- In games: game position, game state, move, game tree/game graph
- Terminal states, result, reward
- Representation of board and moves, internal and external