Computing Science (CMPUT) 455 Search, Knowledge, and Simulations

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Topics:

- Introduction to decision-making
- Optimal decision-making
- · Some models of human decision-making

- TA Office hours began this week
 - See website for times and meeting links
- Start coding Assignment 1
- Quiz 2, review Reinforcement Renaissance
- Read Heingartner, Maybe We Should Leave That Up to the Computer.
- Activities for Lecture 3

Lecture 3: Problem Solving and Decision-making

- What is decision-making?
- Models of the world, reward, and utility
- · How to evaluate alternatives in decision-making?
- Exact evaluation, expected values

- How do humans make decisions?
- Heuristics, Bounded Rationality, and Satisficing
- What is the "right" decision for a program to make?
- Kahneman and Tversky experiments, criticism of utility theory

Decision Making in Humans and Machines



Image source:

blogs-images.forbes.com/mikemyatt/files/

2012/11/decision-making-processes1.jpg

Decision making is studied in many fields

- Business
- Psychology
- Advertising
- Computing Science

• Al

Decision Making in Humans and Machines (2)

- Decision making in politics can have far-reaching consequences (war, peace, prosperity, ...)
- Decision making is big business what to buy, sell, produce,...
- Decision making is studied by many people in many different ways
 - "Common sense"
 - Academic and industry research
 - Popular "how to" books
- We make decisions every day. How and why?

Some big questions:

- Can we make better decisions?
- Can we understand and influence other people's decisions?
- Can we teach decision-making to children, students, employees?
- Can we model decision-making in a computer program?

Decision Theory (Theory of Choice)

Two main strands of research:

- Normative decision theory
 - Analyze decision problem
 - Tell user what is best action
 - Example:

"You should play e4, it is the best move"

Decision Theory (Theory of Choice)

Two main strands of research:

- Normative decision theory
 - Analyze decision problem
 - Tell user what is best action
 - Example: "You should play e4, it is the best move"
- Descriptive decision theory
 - Analyze how real agents (people, programs?) make decisions
 - Example:

"In our user study, 55% played move d4, 32% played e4, and 13% played some other move. The reasons are: ..."

- Classical game theory (e.g. von Neumann and Morgenstern 1947)
- Selfish players, try to maximize their money (*)
- Simplest case: two player zero sum games
- Zero sum my win is your loss
- Actions can involve random outcomes, but with known probabilities
- Goal: maximize expected value

- Concept from probability theory
- Random event, with *n* different outcomes
- Each outcome evaluated by a number value *v_i* (reward, money, ...)
- Probability *p_i* of each outcome known

•
$$\sum_{i=1}^{n} p_i = 1$$

• Expected value (EV):

•
$$\sum_{i=1}^{n} p_i v_i$$

- Throw a six-sided fair die
- Value = the number rolled

• *n* = 6

•
$$p_1 = p_2 = p_3 = p_4 = p_5 = p_6 = 1/6$$

•
$$v_1 = 1, v_2 = 2, v_3 = 3, v_4 = 4, v_5 = 5, v_6 = 6$$

- Expected value
- $\sum_{i=1}^{n} p_i v_i = 1/6(1+2+3+4+5+6) = 21/6 = 3.5$

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- $\sum_{i=1}^{n} p_i v_i = 1/6(1+2+3+4+5+6) = 21/6 = 3.5$
- Question: what is the EV when rolling two dice?

- Assume you play a simple card game, and all you care about is maximizing money
- · Assume you have two possible actions, fold or bid
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- The probability of winning if you bid

- Set p to be your probability of winning if you bid
- Fold: your value is -1.
- Bid: your value is
 - +5 with probability p
 - -3 with probability 1 p
- Which action is better in expectation?

- Bid: +5 with probability p
- Bid: -3 with probability 1 p
- Expected value after bid: 5p + (-3)(1 p) = 8p 3
- When is this better, worse, or equal to -1 (Fold)?
- Depends on p, compare 8p 3 with -1
- When are they equal? Solve equation 8p 3 = -1
- Solution *p* = 1/4

- When p = 1/4, you are *indifferent*
 - *Expected value (EV)* of both actions, fold and bid, is the same
 - Confirm EV for bid:

$$5\times \frac{1}{4} + (-1)\times (1-\frac{1}{4}) = \frac{5}{4} - \frac{9}{4} = -1$$

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- When p grows, 8p 3 also grows
- For p > 1/4, bidding is better
- For p < 1/4, folding is better

Example: when p = 1/3, you want to bid

- EV(fold) = -1
- EV(bid) = $1/3 \times 5 + 2/3 \times -3 = 5/3 6/3 = -1/3$, better than folding

- The analysis before was probably reasonable for most people, describes the "most rational" choice
- What happens if we scale it up?
 - Instead of -1, +5, -3 dollars, play with -10000, 50000, -30000

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- What happens if we scale it up?
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- Question: Is optimizing expected value the "most rational" strategy *now*?

- Some people would hate to lose \$10000 without even trying
- Some people can lose \$10000 without horrible consequences, but not \$30000
- Some people would value winning \$50000 very highly
- Our **utility** of money does not always scale linearly with the amount of money
- It depends on how it affects our life

A paradox about expected value vs. actual behavior of people

- Play a game against the bank:
- The bank puts \$2 in the pot originally
- Each round you flip a coin
- If head, the bank doubles the pot
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- Q1: What is your expected value for this game?
- Q2: How much would you pay to be allowed to play this game?

- Start with \$2
- Head double
- Tail game over
- See Python code petersburg.py, petersburg2.py
- Short demo now.

St. Petersburg Paradox Analysis

- probability 1/2, win \$2 tail
- probability 1/4, win \$4 head, tail
- probability 1/8, win \$8 head, head, tail
- probability 1/16, win \$16 head, head, head, tail
- ..

• Expected value of your win

$$1/2 \times 2$$

 $+1/4 \times 4$
 $+...$
 $= 1 + 1 + ... = \infty$

• How much would you pay to play?

- The expected value is infinity but:
- It includes mostly extremely unlikely events
- Example:
 - Chance of 1/1,024 to win \$1,024
 - Chance of 1/1,048,576 to win \$1,048,576
 - Chance of 1/1,099,511,627,776 to win \$1,099,511,627,776
- How to evaluate those in practice?

- Utility is a concept from economics
- Measures satisfaction of a consumer with an outcome (e.g., receiving a specific good)
- Utility of a good? It determines the price that a consumer is willing to pay
- But what is the utility of money?
- Q: Is twice the money twice as desirable?

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- Q: Is twice the money twice as desirable?
- In general, no.

Utility Function and Risk



Image source: (Shoham & Leyton-Brown, 2008)

- Utility function for money is a mapping:
 - From: Monetary outcome
 - To: utility scale reflecting personal preferences
- Linked with types of behavior:
 - Risk-averse (conservative)
 - Utility function grows slower than linear
 - Risk-neutral
 - Risk-seeking (e.g. playing lottery)
 - Utility function grows faster than linear
Marginal Utility



Image source:

http://s3.crackedcdn.

com/articleimages/dan/

rags/gates3.jpg

- Marginal utility: increase in consumer satisfaction from having one unit more of a good
- Example: what is the value of having \$100 more?
 - Very high if you are broke
 - Very low if you are Bill Gates
 - Marginal utility of money generally decreases with wealth

Car Example



Image source: shedsunlimited.net

Another example:

- What is the marginal utility of owning one more car?
- High if you have no car
- Much lower if you already own 3

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- Q: Do you think people actually act like this?
- Q: Do you think expected utility maximization is necessarily the *best* (most rational) way to act? Why or why not?

Estimating Probabilities, Risk and Insurance

- Insurance companies charge an insurance fee...
- ... in return for promising to reimburse you for a low-probability large loss
- How to come up with a "fair" insurance premium?
- Need to know all the risks bad things that could happen and their probabilities
- Typically, only specific types of risk are covered by a policy
 - Home insurances usually exclude war, water damage, some types of natural and human-made disasters,
- Impossible to estimate singular events, "black swans"

Heuristics



Image source:

http://archimedespalimpsest.

org/images/kaltoon/4.php

- From Greek word "find" or "discover" (wikipedia)
- Any practical problem solving method
- "Mental shortcut", rule of thumb, educated guess, common sense rule, a rough model, using a similar case for guidance, ...

Heuristic vs Exact



Image source:

http://archimedespalimpsest.

org/images/kaltoon/4.php

- Opposites of heuristic approach: exact solution, exhaustive analysis, precise theory
- We rely on heuristics all the time
- Most of life is too complex to "solve exactly"
- Heuristic decision-making and exact methods are both used in computer programs

Heuristics in Computing Science



Image source:

https://stackoverflow.com/

questions/17594924/

- Typical heuristic in CS: solution to simplified problem
- Example problem: find shortest path from A to B
- Real solution: follow roads, avoid obstacles
- Heuristic: straight-line distance

Polya - How to Solve It



Image source:

en.wikipedia.org/wiki/File:

HowToSolveIt.jpg

- A system for human problem-solving
 - Classic book by mathematician George Polya
 - Published in 1945
 - Still popular and influential
- Four principles
- Large set of heuristics

- 1. Understand the problem
- 2. Devise a plan
 - Find connection between data and unknown
- 3. Carry out the plan
- 4. Looking back
 - Examine the solution, review/extend
 - Polya's book focuses on problem-solving mathematics
 - We "translate" some ideas for CS

Polya - How to Solve It - Inside Cover



Principle 1: Understand the Problem

Format: Polya's text in italic - comments below

- What are the data?
 - What is given? What is the input?
- What is the unknown?
 - What is the output?
- What is the condition?
 - What are the requirements/constraints for the solution?

Principle 1: Understand the Problem (continued)

- Draw a figure. Introduce suitable notation.
 - Draw or write down the important concepts in the problem and their relations.
- Separate the various parts of the condition
 - Find smaller parts, functions that make up the required solution

Polya gives a list of general approaches to try. Examples:

- Find connection between data and unknown
 - How do you compute the output as a function of the input?
- Have you seen it before? Do you know a related problem?
 - Can you re-use the previous solution?
- Could you restate the problem?
 - Is there a different way to write it, which is more similar to things you know?

Principle 2: Devise a Plan (continued)

- If you cannot solve the proposed problem try to solve first some related problem
 - Solve a special case
 - Solve a concrete example
 - Drop the complicated parts for now
- Did you use all the data?
 - Are you using everything you know?
 - The whole specification?
 - All properties of the input?

Principle 3: Carry out the Plan

- Carrying out your plan of the solution check each step
 - Write functions to implement your program, test each one separately.
 - Use unit tests to help verify that each function works as expected, at least on the test cases.
- Can you see clearly that the step is correct? Can you prove that it is correct?
 - Use assertions in your code to make sure input and output are as you expect. For really tricky code, you can even try a formal proof with pre- and postconditions and loop invariants (Cmput 204 stuff).

• Can you check the result?

- Examine the solution
- Review the problem in all details, check with your solution
- Can you use the result, or the method, for some other problem?
 - Refactor code, simplify functions, clean up
 - Extend or generalize functions for other problems
 - Organize into modules

- Dictionary of heuristics is largest part of book
- Over 60 entries
- · Some are specific to mathematical problem solving
- Most are generally useful
- Next two slides show examples

Auxiliary problem

- Find an easier problem that will help solve the original
- Example: useful helper function
- Solve problem in several small steps, each implemented in a simpler function

Decomposing and recombining

- Break a big problem into parts
- Find out which parts are important
- Solve parts
- Put together solutions of parts
- Examples:
 - Separate UI from engine
 - Floodfill separate scan of full board from what to do in each area
 - Separate tree search algorithm from details of what to do in each node

Herb Simon and Bounded Rationality



Image source:

http://www.cs.cmu.

edu/simon/

- Herb Simon (1916 2001)
 - One of founders of AI (and other disciplines)
 - Nobel-prize winner
 - Professor at Carnegie-Mellon
- Original background: decision-making in business, economics
- Criticized "perfect rationality" assumption of previous theorists
- Developed influential concept of "Bounded Rationality"

See activities course page

- Video 1: The Limits or Bounds of Rationality
- Video 2: What is Intuition?
- Optional read more about Herb Simon: https: //en.wikipedia.org/wiki/Herbert_A._Simon

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 Much harder example: what should my company produce to maximize its profit?

- Mathematical optimization: find the **best possible** solution to a problem
- Simple example: what is the minimum of function $x^2 - 5x + 3$?
- Harder example:

How many regular size soccer balls can we pack into a standard shipping container?

- Much harder example: what should my company produce to maximize its profit?
- Even harder: what should the company produce to make the most people the happiest?

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- No:
 - Decision involves many factors that are hard to compare
- Example:

study more, or get more sleep?

Can We Make Perfect Decisions?

- Herb Simon:
 - Most often, no.
- Why?
- Humans (and computers) have:
 - Limited memory
 - Limited time to make a decision
 - Incomplete, or wrong, information about actions and results
 - Limited powers of logic, deduction, lookahead
 - Limited imagination to come up with new approaches
 - Limited everything ...

Bounded Rationality - Discussion

- Perfect decisions often not possible in practice
- How can we act well, while acknowledging our limitations?
- How can we use what we know, and even what we don't know?
- How do we deal with multiple, conflicting goals?
- When should we use heuristics, and when a more systematic search?
- What is the "best" thing to do, given our limitations? Is that even well-defined?
Exact vs Good Enough, Satisficing

- Humans use heuristics as shortcuts
- Concept of satisficing (Herb Simon)
- Trying to optimize is often too hard
- More reasonable:
 - Define criteria for "good enough"
- A satisficing solution is one that fulfills these criteria
- Example in games: play a "good" or "strong" move,

even if we cannot prove it is the best

"Decision makers can satisfice either by finding optimum solutions for a simplified world, or by finding satisfactory solutions for a more realistic world. Neither approach, in general, dominates the other, and both have continued to co-exist..."

Optimum Solutions for a Simplified World



Image source: www.

simpleitsolutions.com

- Example: what is the cost of buying a new computer?
- Simple answer: look at the price tag
- More complete answer: add tax, cost of new software, cost of time for upgrades, electricity, insurance, carrying bag, ...
- In practice, we ignore or roughly summarize many of these details and make a decision for a simplified problem
- Some costs are not known anyway, e.g. future costs of electricity, repairs, ...

Satisfactory solutions for a more realistic world



Image source:

thehealthysubstitute.

wordpress.com

- Example: what to eat for lunch?
- A myriad of choices
- Many small or large variations are possible (seasoning, extras, ...)
- In real life, we only consider a small number of choices
- We (usually) satisfice, not optimize

Comment - Model vs Direct Observation

• Remember Herb Simon's quote:

Decision makers can satisfice either by finding optimum solutions for a simplified world, or by finding satisfactory solutions for a more realistic world.

- Two approaches to reasoning for decision-making:
- Reasoning based on a model of the world
- Reasoning from direct observations of the world
- Big topic in Reinforcement Learning
- In games, we have a *perfect* model. But that is the exception, not true for real world

Kahneman and Tversky



Image source: www.vanityfair.com/

news/2016/11/

- Very influential psychologists
- Kahneman won Nobel prize in economics
- Humans have systematic cognitive biases
- Most are averse to loss and ambiguity, "losses loom larger than gains"
- Activity: Watch Daniel Kahneman Videos

Kahneman and Tversky - Anchoring

Reg \$48 SALE

Image source: https:

//www.jeremysaid.com/blog/

anchoring-effect-power-conversion-optimization/

- People tend to "anchor" on first impressions
- Later decisions made relative to this, not in absolute terms
- People focus more on *changes* in their utility than on *absolute* utilities

Anchoring - Another Car Example

Scenario A

- Monday, I offer to sell you my car for \$30000.
- Tuesday, I offer it to you for \$20000.

Scenario A

- Monday, I offer to sell you my car for \$30000.
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Scenario B (same car)

- Monday, I offer to sell you my car for \$10000.
- Tuesday, I offer it to you for \$20000.

Scenario A

- Monday, I offer to sell you my car for \$30000.
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Scenario B (same car)

- Monday, I offer to sell you my car for \$10000.
- Tuesday, I offer it to you for \$20000.

Question: In which scenario are you more likely to accept the offer on Tuesday?

- Quick tour of theories and experiments in human decision-making
- How do we make decisions?
- · Limits to making "perfect" decisions
- Bounded rationality and satisficing
- Expected value, expected utility
- Cognitive biases
- Next time:
 - Formal models of decision-making in sequential games
 - Representing games