Computing Science (CMPUT) 455

Search, Knowledge, and Simulations

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Today's Topics - Lecture 11

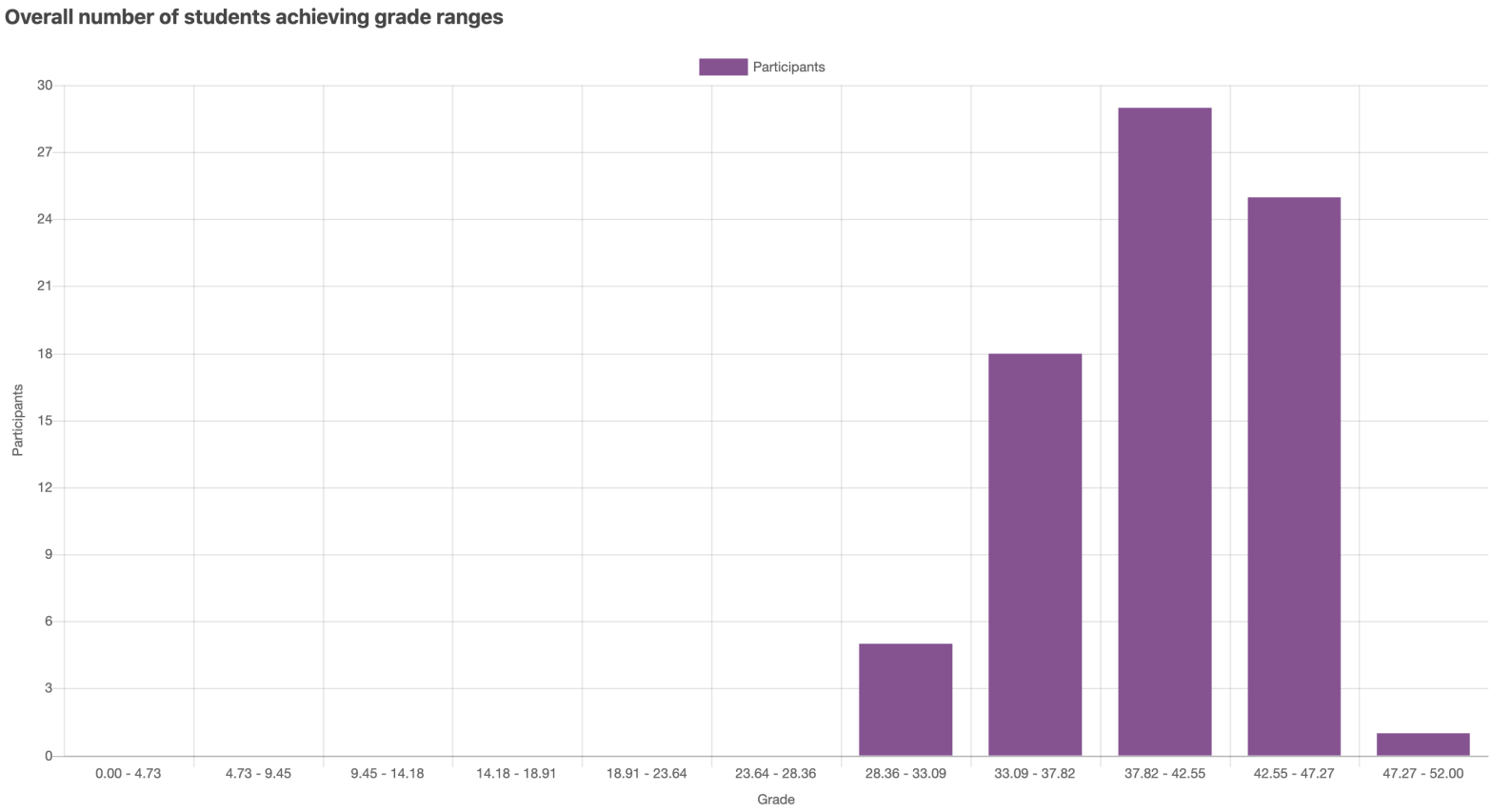
- Midterm results
- Quiz 5 review
- Quiz 6 review
- Defining and using knowledge in heuristic search
- State evaluation
- Move evaluation

Coursework

- Assignment 2: due Monday, Oct 18 (i.e.,next Monday!)
- Quiz 7: due Monday, Oct 25 (i.e., *not* next Monday)
  - Topic: Simulations (lecture 12)
- Read van der Werf et al., *Solving Go on small boards*

Midterm Statistics

- 78 attempts, average 77.7%



Won't review questions:

- In-class: Everyone got different questions
- eClass: We reuse some questions from year to year

Quiz 5 Review

- Quiz 5, Minimax search
- 91 (!) attempts. Average grade: 87%
- Lowest scores: Q15: 72.5%, Q16: 56%

Quiz 5 Review: Q15

Q15 For games with only two outcomes (win-loss), how do boolean negamax and standard alphabeta search as in alphabeta.py compare in terms of pruning power? Assume that in alphabeta, a win is represented by +1 and a loss by -1.

- A Boolean negamax can prune more
- B Alphabeta can prune more
- C They are exactly the same, when the search is started with a full window (-infinity, +infinity)
- D They can be made the same by using a different alphabeta window

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- *A*: At the first MAX node, alphabeta has to evaluate every child before it learns that the maximum possible value is 1 (i.e., it will have to keep looking after it finds a 1)
  - Boolean negamax can stop right away when it finds TRUE

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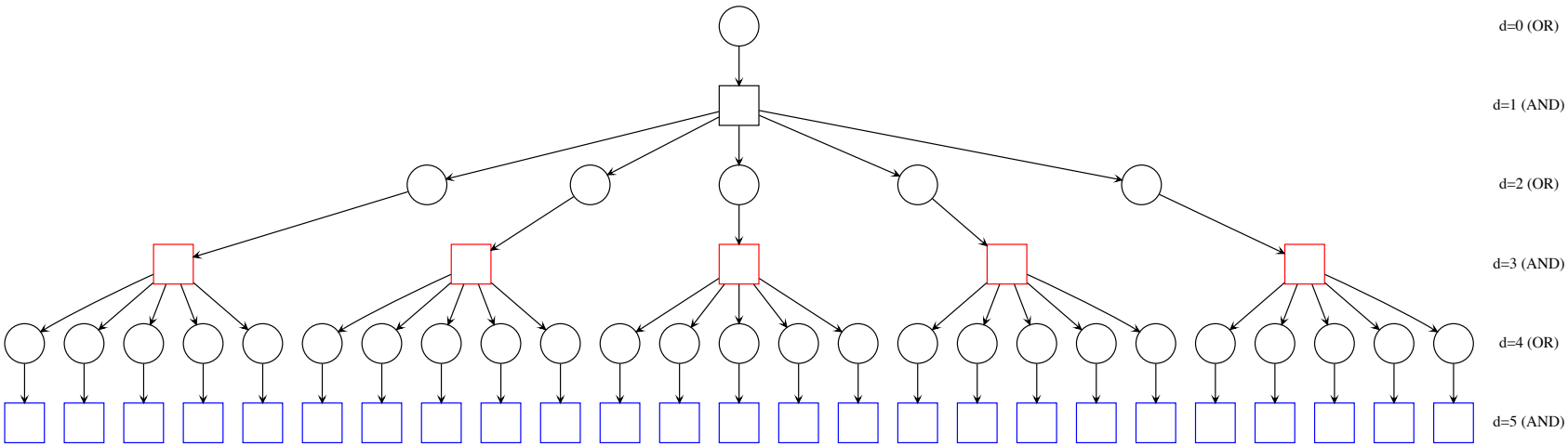
  - Boolean negamax can stop right away when it finds TRUE
- But also *D*: If we initialize the window to $(-1, 1)$, then alphabeta can prune immediately

Quiz 5 Review: Q16

Q16 Search 1: We use boolean negamax search to solve a game with of constant branching factor $b = 5$ and constant depth $d = 10$. Assume that the game is a win for the first player and that our program has perfect move ordering. Let n1 be the number of nodes searched.
Search 2: after we further improve our program, it can now solve the same game while searching to a depth of only 8. Let n2 be the number of nodes searched now.
Question: Approximately, what is the ratio n1/n2?

- A 5 B 1 C 2 D 8 E 10 F 25 G >100
- There are $b^2 = 25$ fewer paths at depth 8 than at depth 10, *BUT*
- Perfect ordering means that only a single branch needs to be searched from each OR node
- So one of the two "saved" levels will have exactly 1 path from each node
- *A*: So there will only be $b = 5$ times fewer paths searched

Quiz 5 Review: Q16 cont.



Example: search to $d = 3$ (red) instead of $d = 5$ (blue)

- $5\times$ more nodes at $d = 4$ than $d = 3$
  - Why? Each AND must search all 5 children
- *Same number* of nodes at $d = 5$ as at $d = 4$, because $d = 4$ is OR nodes
  - Each OR node searches only a *single* child (due to perfect ordering)

Quiz 6 Review

- 74 attempts, average 90.2%
- Lowest scores: Q17: 79.7%, Q18: 75.7%

Quiz 6 Review: Q17

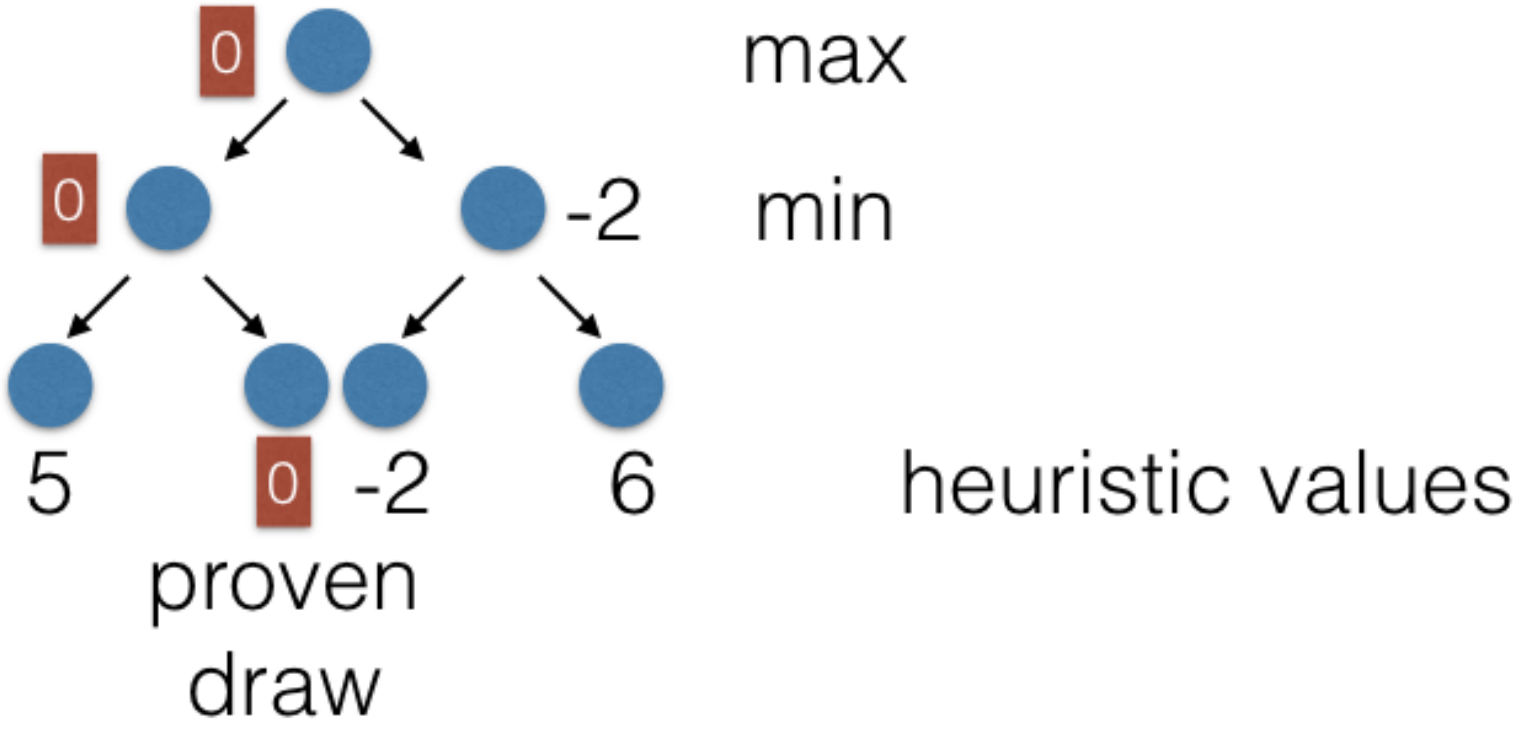
Q17 Assume we search a chess position with an evaluation function that can return either exact or heuristic values. We use exact values for win = 1000, draw = 0, loss = -1000 for terminal states, and other numbers (not equal to these) for non-terminal states. The result of a depth-limited alphabeta search is 0. Claim: this proves that the game is a draw.

- Answer: *False*
- We did a depth-limited search
- Some “leaves” were evaluated using the heuristic

Quiz 6 Question 17 Review

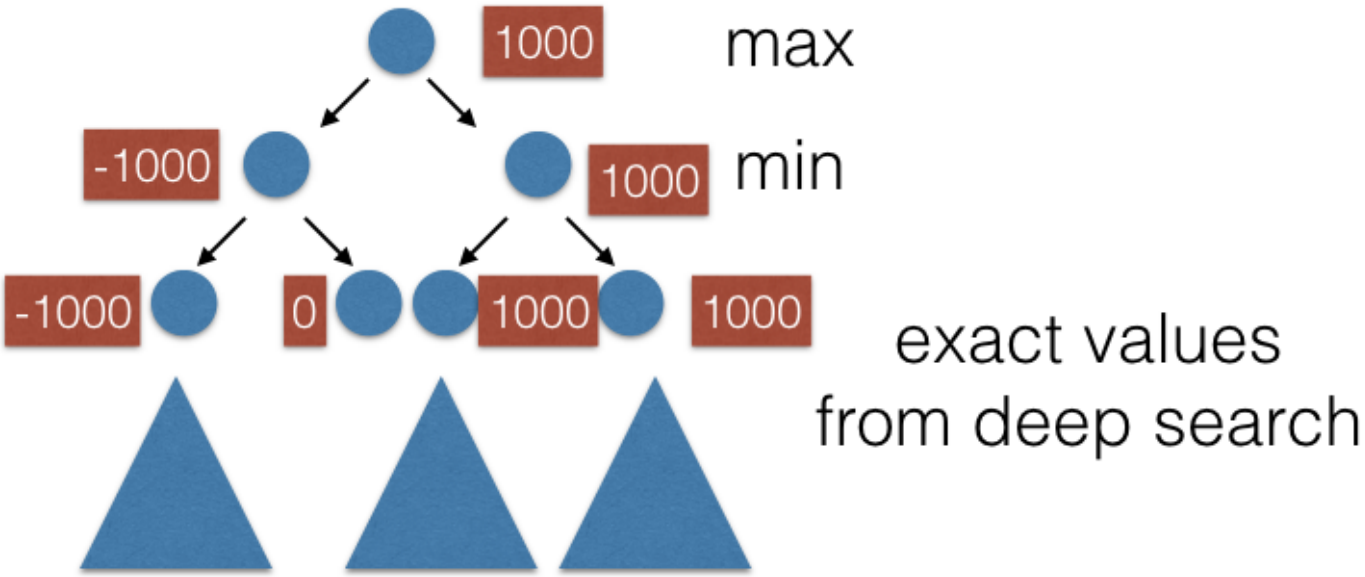
- We did a depth-limited search. Some “leaves” were evaluated using the heuristic.
- Search establishes two proofs, one for each player
  - heuristic value of root ≥ 0 for max
  - heuristic value of root ≤ 0 for min
- So it is not a proven draw
- With deeper search, the values of some of those “leaf” nodes will change
- Some could turn out to be wins or losses
- This can **change** the minimax value!
- See examples next slides

Quiz 6 Question 17



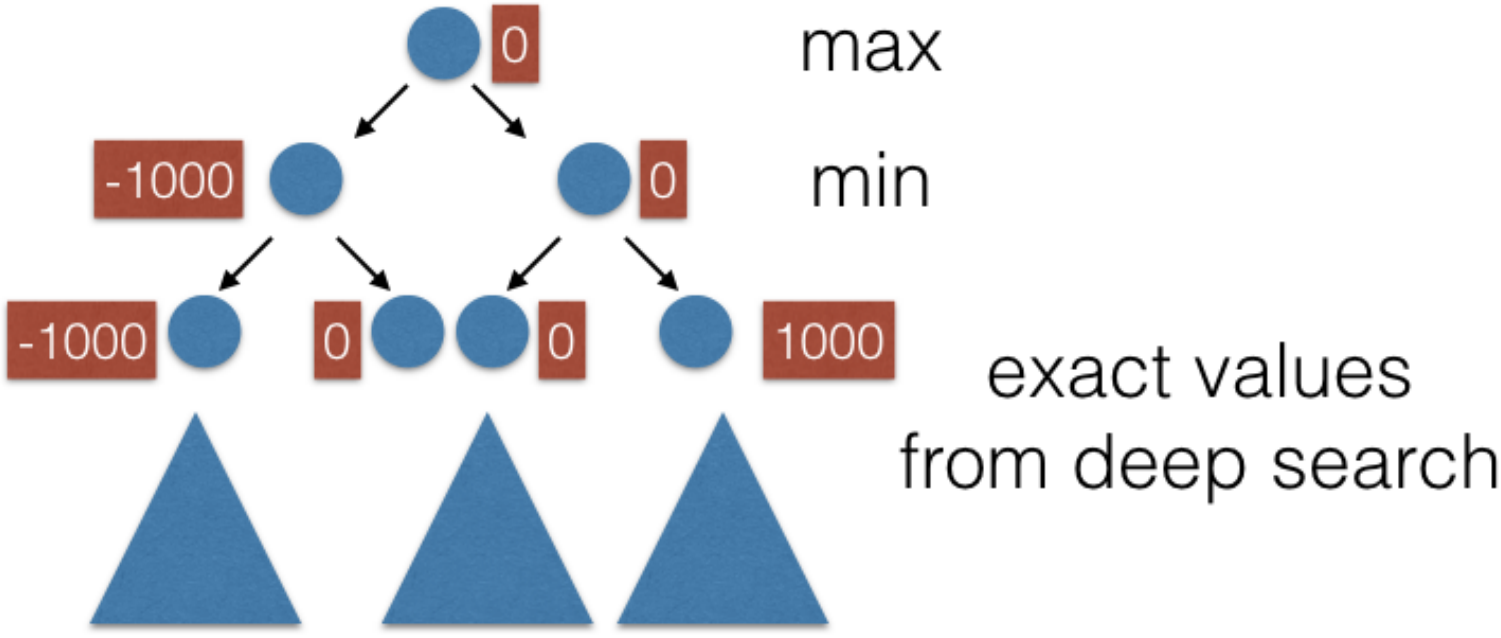
- Leaf that is proven draw gives minimax score
- Other leaves evaluated by heuristic

Quiz 6 Question 17



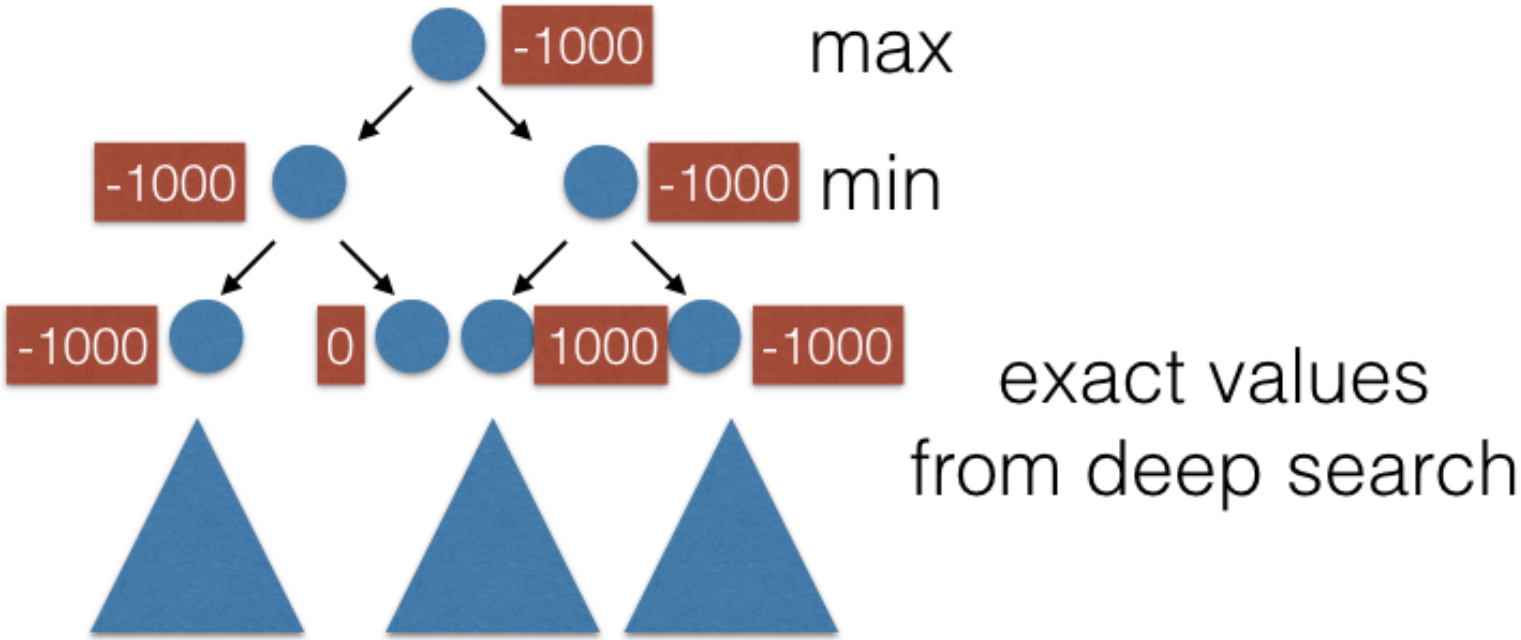
- Deeper search proves true value of other leaves
- Root becomes a win

Quiz 6 Question 17



- Another possible outcome: Root becomes a draw

Quiz 6 Question 17



- Another possible outcome: Root becomes a loss

Quiz 6 Question 18 Review

- Question 18: We do a depth-limited alphabeta search of a checkers position, with a heuristic evaluation function for this game. Assume that all heuristic evaluation scores are larger than proven-loss and smaller than proven-win. The search does not always reach the end of the game. It returns a proven-draw minimax score. Claim: we can trust this result, the position must be a draw.
- Answer: False
- The result depends on the heuristics used to evaluate other nodes. So we cannot trust it. For example, there could be a win that is evaluated by a negative heuristic score. Then the program would not have that move in its PV.

Knowledge in Heuristic Search

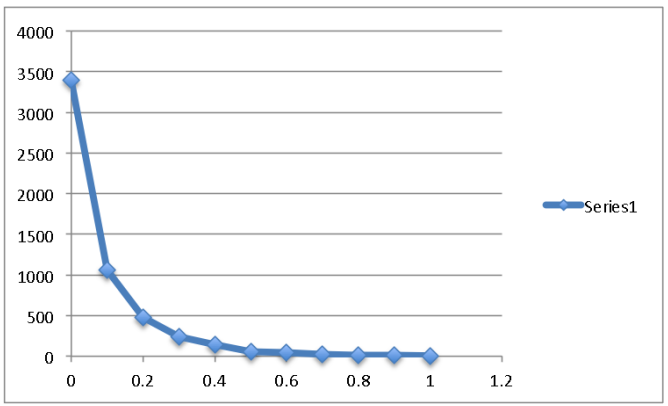
Using Knowledge in Heuristic Search

- How can knowledge be used in heuristic search?
  - Evaluate states
  - Evaluate actions
  - Other ways
- Properties and interpretations of knowledge for heuristic search
  - Probabilities, preferences, ordering, ranking
- Representing knowledge
  - Rules, patterns, features, neural nets
- Acquiring knowledge
  - Manual vs machine learning

Knowledge for Search - the Story So Far

- Discussed many search techniques
- Most were “knowledge-free”
  - Blind search
  - Random sampling
  - Solving games with boolean negamax or alphabeta
- Others used a “black box” heuristic evaluation function
  - Depth-limited alphabeta search
  - Move ordering heuristic
- How to build such a heuristic function?

Treasure Hunt Example from Lecture 7



- Results from running `heuristic_search_on_tree.py`
- A little bit of heuristic guidance greatly reduced number of samples until treasure was found
- With perfect heuristic: walk straight to goal
- Question: Could we do as well in games with a perfect heuristic?

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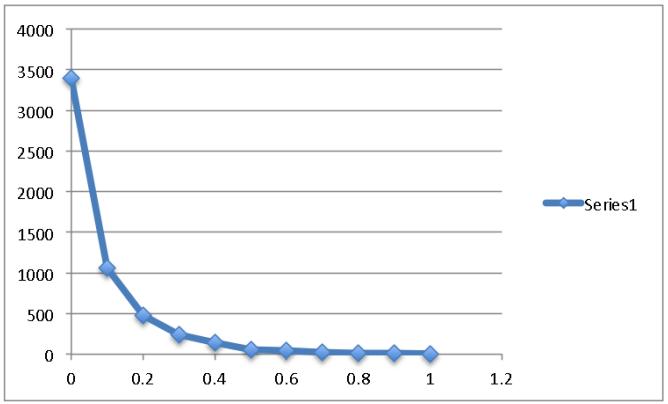


- Results from running `heuristic_search_on_tree.py`
- A little bit of heuristic guidance greatly reduced number of samples until treasure was found
- With perfect heuristic: walk straight to goal
- Question: Could we do as well in games with a perfect heuristic?
- *Not quite.* Remember proof trees. Even with perfect heuristics, need to cover *all* the opponent's move choices.
  - Not sufficient to find a single path to the "goal"

Big Questions - Knowledge for Heuristic Search

- What is knowledge used for?
- Where does it come from?
- How is it selected?
- How is it constructed?
- How is it learned?

Kinds of Knowledge in Heuristic Search

- Many kinds of knowledge in heuristic search
- The “big two” for us:
  1. State evaluation
  2. Move evaluation
- Many other examples of using knowledge

Other Kinds of Knowledge in Heuristic Search

- Time control, search depth control
- Knowledge to reduce size of state space
  - DAG vs tree
  - Benson's algorithm in Go
- Efficient state representation (we discussed)
- Knowledge about algorithm optimization and tuning
- Many more...

Knowledge for State Evaluation

State evaluation

- What is it?
  - Mapping from (full) state to one number
- What does it measure?
  - How good is that state?
- Other terms with similar meaning:
  - Evaluation function
  - Position evaluation

Knowledge for Move Evaluation

Move evaluation

- What is it?
  - Mapping from move (action) to number
- What does it measure?
  - How good is that move?
  - Example: probability that this is the best move
- Can be used as filter:
  - Which moves are (probably) bad and should be filtered out (pruned)
- Other terms with similar meaning:
  - Action value
  - Move value
  - Q-value in machine learning

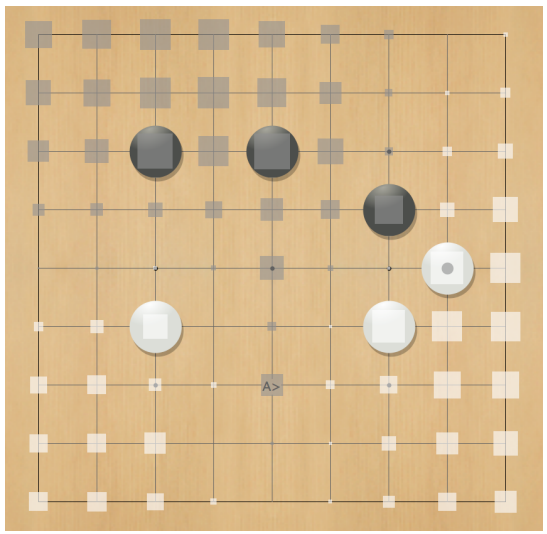
Details on State Evaluation

- We know exact evaluation in **terminal states**
  - Win, loss, draw, win by 23.5 points,...
- What about heuristic evaluation in **non-terminal states**?
- In games, two kinds of evaluation are popular
- Heuristic evaluation:
  - Higher is better
  - “Just a number”, no extra meaning, used for ranking/ordering moves
- Winning probability:
  - Higher is better
  - Has an interpretation as probability

Heuristic State Evaluation in Search

- Most important use of state evaluation:
  - Evaluation function in search
- Leaf nodes *of search* evaluated by this function
  - Exact evaluation for terminal states
  - **Heuristic evaluation of non-terminal states**
- Interior nodes in search evaluated by minimax rule
  - Backup the evaluation in leaf nodes:
    - to parent . . .
    - to grandparent, further up . . .
    - . . . all the way to the root

Heuristic Evaluation Function



- Heuristic evaluation:
  higher is better
- One possible interpretation:
  estimate of score of game
  - +12
  - “Black is about 12 points ahead”
- May have no “hidden” interpretation
  - Evaluation can be “just a number”
  - Used just for relative ranking of positions

Example: Evaluation Function in Chess

- Queen = 9, rook = 5, bishop = 3,...
- Positional features, such as location of pieces
- Evaluation =
  sum of my material's values
  - sum of opponent's material's values
- Higher value is better
- No "deeper meaning", just a number
- Use in search, example:
  - Move A leads to position with evaluation 1.49
  - Move B leads to position with evaluation 1.52
  - Search will select B over A

Relative vs Absolute Evaluation

- For decision-making, the evaluation numbers themselves do not matter
- Only the *ordering* given by the numbers matters (higher is better)
- It decides the *preference* or *ranking* between moves
  - Example 1: multiply all evaluations by 10
  - Example 2: add 7 to all evaluations
  - $\Rightarrow$ The search will make exactly the same decisions

Relative vs Absolute Evaluation

- Monotonically increasing (order-preserving) function:

$$x > y \quad \Longrightarrow \quad f(x) > f(y)$$

- Any monotonically increasing mapping of the evaluation function will give the same search behavior
- Same idea in utility theory (Lecture 3): *ordinal utility*

Relative vs Absolute Evaluation

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- Any monotonically increasing mapping of the evaluation function will give the same search behavior
- Same idea in utility theory (Lecture 3): *ordinal utility*
- Cardinal comparisons (exact values) become important when you aggregate multiple evaluations
  - E.g., by taking expectations
  - Some increasing mappings still give same search behavior when evaluations aggregated
  - Others might not

But - What Does an Evaluation Mean?

One Interpretation:

- General motto:
  "Similar evaluation values for similar states"

More precise version

- All states with the same evaluation are "equally good"
- They have the same (unknown to us)
  probability of winning
- Higher evaluation = higher probability of winning
- Evaluation function partitions set of all states S
  - Subsets S_v, one subset for each value v
  - All states in same S_v are assumed equally good for us

Mixing Exact and Heuristic Evaluation

- We can mix exact and heuristic evaluations
- If we are careful, we can get true proofs of wins and losses this way
- Example: win = 10000, highest heuristic score = 5000
- If alphabeta returns 10000, it is a proven win
- Having a good heuristic can help speed up an exact proof
  - Provides good move ordering for iterative deepening search
  - A better move sorted first means more cuts in the tree search
- Careful: remember problem with draw vs heuristic scores in proving TicTacToe

What does Winning Probability Mean?

- Different interpretations
- Clearest case:
  - game with chance element
  - e.g. dice rolls
- The winning probability **is** the minimax score!
- Example from backgammon
  - Endgame state s
  - Need to roll two sixes to win, lose otherwise
  - Probability of rolling two sixes = $1/6 \times 1/6 = 1/36$
  - State evaluation $v(s)$ = 1/36
  - Use as evaluation in search:
    move to s if it has the highest winning probability

Winning Probability in Games with No Chance

- There are no probabilities in the game itself
- A perfect player would know with certainty
  - True winning probability is either 0% or 100%
- Probability can arise from
  - Our imperfect understanding of the game
  - Generalization and abstraction in our evaluation
  - Randomness in our strategy
  - Randomness or imperfect understanding of the opponent's strategy

Example - Winning Probabilities in Value Net

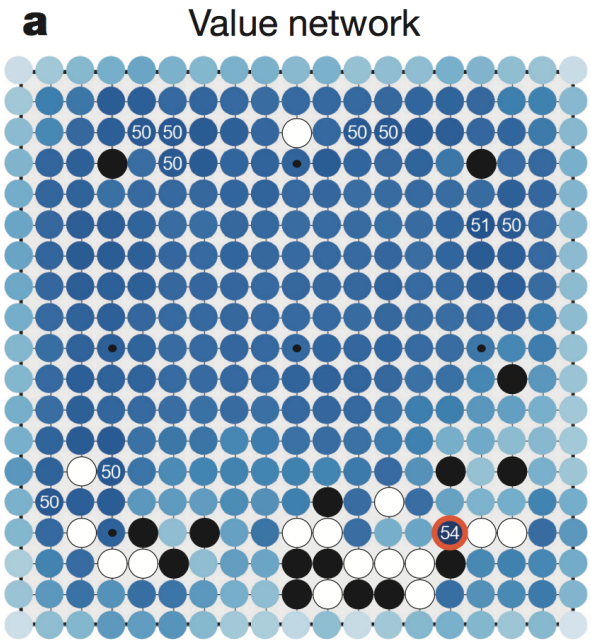


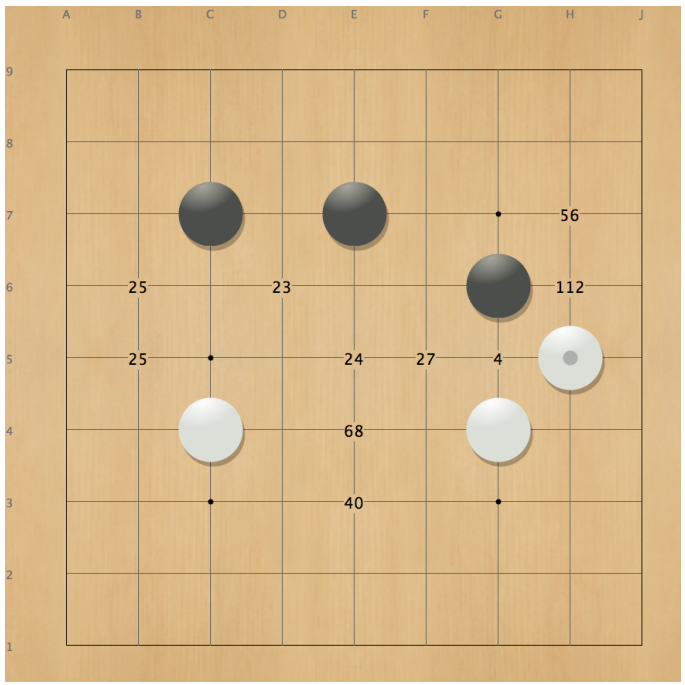
Image source: Silver et al, Mastering the game of Go with deep neural networks and tree search, Nature

- Can use machine learning to *learn estimates of win probabilities*
- Example: AlphaGo's value network
- Deep neural net
- Map: from state to learned win probability

Using Knowledge for Move Evaluation

- Given a state
- Also given the possible moves from that state
- Put a numeric value on each move
- Main use: action selection in search, in simulation
- Can also be used for
  - Move ordering in search
  - Move pruning
- Again, we can have both types of evaluation
  - Without a probabilistic interpretation
  - With a probabilistic interpretation

Move Evaluation as “A Number”



- No interpretation
- Bigger is better
- Example: Go program Explorer (ca. 1989 - 1995)
- Where did its evaluations come from?
- Large number of hand-made heuristics for different types of moves

Move Evaluation as Probability

- Assume move i has probability p_i:
- Interpretation 1:
  - p_i is probability that move i is a win
- Interpretation 2:
  - p_i is probability that move i is the best move
- Both make sense
- Which one you use depends on
  how you compute or estimate those numbers

Relation between State and Move Evaluation (1)

- Case 1:
- We have state evaluation
- We need move evaluation
- Easy - do a 1 ply search
- Evaluation of move:
  - Evaluation of state after making that move
- Main disadvantage: slow if branching factor is large
  - Example - 19 $\times$ 19 Go: over 300 moves, evaluations, undo to make a single decision

Relation between State and Move Evaluation (2)

- Case 2:
- We have only move evaluation
- We need state evaluation
- No easy solution
- We could try to do “greedy rollout” by following the sequence of best-evaluated moves
- Slow, noisy result
- Not used in practice (compare with randomized simulations later)
- Still have to evaluate the terminal state to get a value

Acquiring Evaluation Knowledge

- Where do evaluations come from?
- (now) Machine learning
- (old) Local goal-directed search
- (old) Handcoded rules
- First, discuss how to represent knowledge in a program

Representing Knowledge for Evaluation

- Many ways to represent knowledge
- Handcoded rules
- Simple features
- Pattern databases
- Neural nets

Handcoded Rules

```
def selfatari(board, move, color):
    maxoldliberty = maxliberty(board, move,
                               color, 2)
    if maxoldliberty > 2:
        return False
    cboard = board.copy()
    isLegal = cboard.move(move, color)
    if isLegal:
        newliberty = cboard.liberty(move, color)
        if newliberty == 1:
            return True
    return False
```

- Most direct way
- Example: heuristic move filter, “avoid selfatari”

Simple Features

```
enum FeBasicFeature{
FE_PASS_NEW,
FE_PASS_CONSECUTIVE,
FE_CAPTURE_ADJ_ATARI,
...
FE_CAPTURE_MULTIPLE,
FE_EXTENSION_NOT_LADDER,
FE_EXTENSION_LADDER,
...
FE_TWO_LIB_SAVE_LADDER,
FE_TWO_LIB_STILL_LADDER,
...
FE_SELFATARI,
FE_ATARI_LADDER,
...
FE_DOUBLE_ATARI,
FE_DOUBLE_ATARI_DEFEND,
FE_LINE_1,
FE_LINE_2,
FE_LINE_3,
...
}
```

- Idea: each feature is a boolean statement about a state, or a move
- Each feature is simple and easy to compute
- With machine learning, we can construct an evaluation function from a combination of many simple features
- Move feature vector: (0,0,1,...,1,1,0,...,1,0,...0,0,...)
- Examples: next few slides

Remi Coulom's Simple Features (1)

Feature	Level	γ	Description
Pass	1	0.17	Previous move is not a pass
	2	24.37	Previous move is a pass
Capture	1	30.68	String contiguous to new string in atari
	2	0.53	Re-capture previous move
	3	2.88	Prevent connection to previous move
	4	3.43	String not in a ladder
	5	0.30	String in a ladder
Extension	1	11.37	New atari, not in a ladder
	2	0.70	New atari, in a ladder
Self-atari	1	0.06	
Atari	1	1.58	Ladder atari
	2	10.24	Atari when there is a ko
	3	1.70	Other atari
Distance to border	1	0.89	
	2	1.49	
	3	1.75	
	4	1.28	

Source: Remi Coulom, Computing Elo Ratings of Move Patterns in the Game of Go

Remi Coulom's Simple Features (2)

Distance to previous move	2	4.32	$d(\delta x, \delta y) = \lvert\delta x\rvert + \lvert\delta y\rvert + \max(\lvert\delta x\rvert, \lvert\delta y\rvert)$
	3	2.84	
	4	2.22	
	5	1.58	
	...	...	
	16	0.33	
	≥ 17	0.21	
Distance to the move before the previous move	2	3.08	
	3	2.38	
	4	2.27	
	5	1.68	
	...	...	
	16	0.66	
	≥ 17	0.70	

Source: Remi Coulom, Computing Elo Ratings of Move Patterns in the Game of Go

Pattern Databases

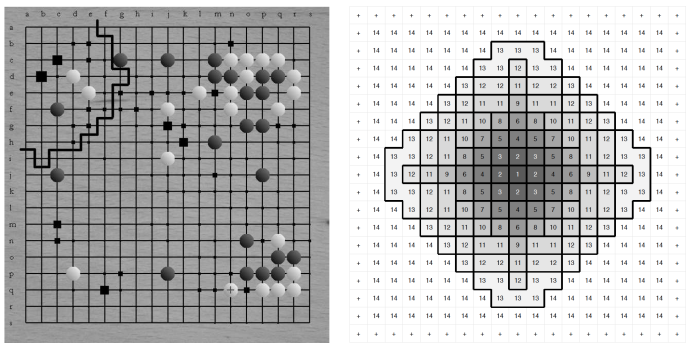


Image source: Stern et al, Bayesian Pattern Ranking for Move Prediction in the Game of Go

- Large patterns can be learned from master games, if they are frequently used
- In Go, typically we have many different sizes of pattern:
- From small 3x3 patterns to full board
- A main question is how to evaluate such patterns
- Measure how often the move in the center is played immediately, or later

Neural Nets

Convolutional Neural Network (CNN)

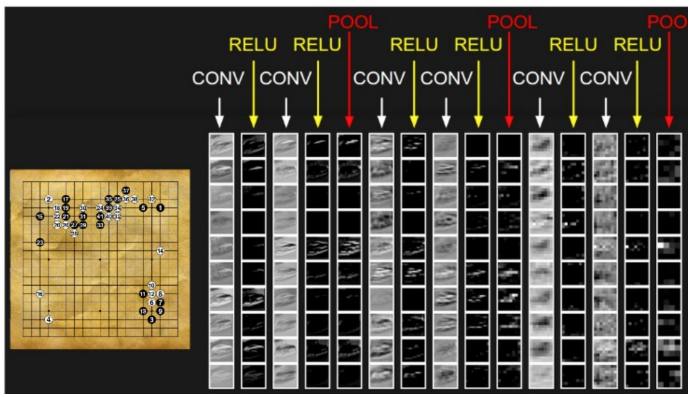
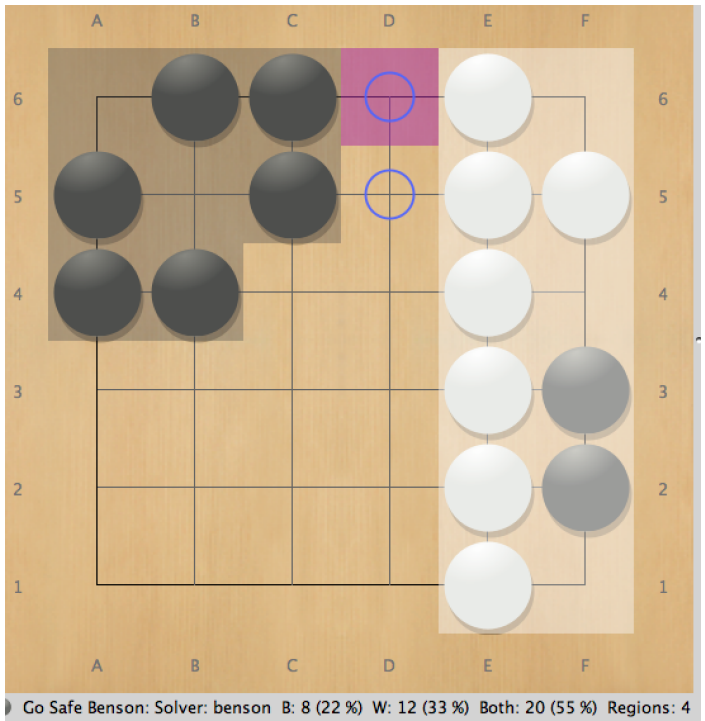


Image source:

`https://www.slideshare.net/ShaneSeungwhanMoon/how-alphago-works`

- Represent knowledge in (large number of) weights of the neural net
- Lower levels of net encode local knowledge (e.g. 3x3, 5x5)
- Higher levels can express global evaluation
- Much more on nets later in the course

Example of Exact Knowledge: Benson's Algorithm in Go



- Benson's algorithm finds stones and territories that are *unconditionally alive*
- No matter what opponent plays: they can never capture our stones
- A generalization of "two eyes"
- Can be used as a filter in a program - do not generate moves in safe territory

Summary

- Many kinds of knowledge
- Used for evaluating states and moves
- Heuristic rules, patterns, neural networks
- Exact knowledge, e.g. safe stones