

# Game Theory and Voter Turnout

By

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# Motivation

- Election Turnout Prediction
- Understand people's motivations
- How to encourage people to turn up to vote
- Indian Election – April to may – 900 Million eligible voters
- Create scalable models for such numbers

## Economic Theory of Political Action in a Democracy - Anthony Downs (1957)

- Assumption:
  - Universal Suffrage
  - Two or more Parties
  - Voters' utilities are a function of govt. action
  - Govt.'s policies are a function of popular desires and opposition policies
  - Opposition Party's policies are a function of govt's policies and people's utility income from incumbent's actions
  - Parties' sole purpose is to get elected

# Economic Theory of Political Action in a Democracy (contd..)

- Two scenarios:
  - Perfect Information
  - Imperfect Information
- Perfect Knowledge:
  - Voters know the govt.'s and opposition's policy function
  - Govt. and Opposition know voters' utility functions
- Imperfect Knowledge
  - Different entities have varying amounts of information
  - Voters might not know about all actions taken by the govt.
  - Voters might not know the govt.'s and opposition's policy function

# Economic Theory of Political Action in a Democracy (contd..)

- Imperfect Knowledge
  - Some individuals will have more information than others
  - Individuals with less information can be swayed by those who have more information
  - Information is costly (time)
  - Voters are rational => Information is gathered only if Marginal expected utility of additional unit of information is greater than the Marginal expected cost
  - Marginal utility of additional information is the expected utility that will be received if the voter votes “correctly” instead of “incorrectly”
- Conclusion: Individual voter’s returns from voting “correctly” are infinitesimal. It is not rational to vote since that voter’s vote is not likely to be pivotal

# Critique

- Number of eligible voters voting are  $\gg 0$
- Model does not take into consideration, the intrinsic utility of the act of voting

# The paradox of voter participation? A Laboratory Study - DAVID K. LEVINE and THOMAS R. PALFREY (2005)

- Participation (Voting) Game:
  - Two parties – A and B
  - $N_A$  ,  $N_B$  and  $f(\cdot)$

Table 1: Expected payoff matrix for individual  $i$  of group  $A$ .

|                           | Vote                  | Abstain         |
|---------------------------|-----------------------|-----------------|
| $n_A^{-i} > n_B^{-i} + 1$ | $H - c_i$             | $H$             |
| $n_A^{-i} = n_B^{-i} + 1$ | $H - c_i$             | $H$             |
| $n_A^{-i} = n_B^{-i}$     | $H - c_i$             | $\frac{H+L}{2}$ |
| $n_A^{-i} = n_B^{-i} - 1$ | $\frac{H+L}{2} - c_i$ | $L$             |
| $n_A^{-i} < n_B^{-i} - 1$ | $L - c_i$             | $L$             |

Reference: Herrmann O, Jong-A-Pin R, Schoonbeek L. A prospect-theory model of voter turnout.

# The paradox of voter participation? A Laboratory Study

$$P_{A,break}^* = Prob(\text{voter in group } A \text{ breaks a tie})$$

$$= \sum_{k=0}^{N_A-1} \binom{N_A-1}{k} \binom{N_B}{k} (p_A^*)^k (1-p_A^*)^{N_A-1-k} (p_B^*)^k (1-p_B^*)^{N_B-k},$$

$$P_{A,create}^* = Prob(\text{voter in group } A \text{ creates a tie})$$

$$= \sum_{k=0}^{N_A-1} \binom{N_A-1}{k} \binom{N_B}{k+1} (p_A^*)^k (1-p_A^*)^{N_A-1-k} (p_B^*)^{k+1} (1-p_B^*)^{N_B-1-k},$$

$$P_{B,break}^* = Prob(\text{voter in group } B \text{ breaks a tie})$$

$$= \sum_{k=0}^{N_A} \binom{N_A}{k} \binom{N_B-1}{k} (p_A^*)^k (1-p_A^*)^{N_A-k} (p_B^*)^k (1-p_B^*)^{N_B-1-k},$$

$$P_{B,create}^* = Prob(\text{voter in group } B \text{ creates a tie})$$

$$= \sum_{k=0}^{N_A-1} \binom{N_A}{k+1} \binom{N_B-1}{k} (p_A^*)^{k+1} (1-p_A^*)^{N_A-1-k} (p_B^*)^k (1-p_B^*)^{N_B-1-k}$$



# The paradox of voter participation? A Laboratory Study

- Size effect – Voter turn out reduces as Total eligible turnout increases
- Competition effect – Turnout expected to be higher in elections expected to be closer
- Underdog effect – The turnout is more for the candidate with fewer supporters
- Experiments:
  - Only varied  $N_A$  and  $N_B$  .  $f$  is fixed
  - $N \in \{3, 9, 27, 51\}$
  - For each electorate size (landslide)  $N_B = 2 N_A$  and (tossup)  $N_B = N_A + 1$
  - $f =$  uniform distribution from 0 to 55

# Predicted Outcomes

| $N$ | $N_A$ | $N_B$ | No. of<br>Subjects | No. of<br>Sessions | $P^*_A$ | $P^*_B$ |
|-----|-------|-------|--------------------|--------------------|---------|---------|
| 3   | 1     | 2     | 51                 | 4                  | .537    | .640    |
| 9   | 3     | 6     | 81                 | 9                  | .413    | .375    |
| 9   | 4     | 5     | 81                 | 9                  | .460    | .452    |
| 27  | 9     | 18    | 108                | 4                  | .270    | .228    |
| 27  | 13    | 14    | 108                | 4                  | .302    | .297    |
| 51  | 17    | 34    | 102                | 2                  | .206    | .171    |
| 51  | 25    | 24    | 102                | 2                  | .238    | .235    |

# Actual Outcomes

| $N$ | $N_A$ | $N_B$ | $\hat{p}_A$ | $p_A^*$ | $\hat{p}_B$ | $p_B^*$ |
|-----|-------|-------|-------------|---------|-------------|---------|
| 3   | 1     | 2     | .539 (.017) | .537    | .573 (.012) | .640    |
| 9   | 3     | 6     | .436 (.013) | .413    | .398 (.009) | .374    |
| 9   | 4     | 5     | .479 (.012) | .460    | .451 (.010) | .452    |
| 27  | 9     | 18    | .377 (.011) | .270    | .282 (.007) | .228    |
| 27  | 13    | 14    | .385 (.009) | .302    | .356 (.009) | .297    |
| 51  | 17    | 34    | .333 (.011) | .206    | .266 (.008) | .171    |
| 51  | 25    | 26    | .390 (.010) | .238    | .362 (.009) | .235    |

# Behavioral Model of Turnout -Jonathan Bendor, Daniel Diermeier, Michael Ting (2003)

- Non voters – Shirkers
- $n_D$  and  $n_R$
- $I \in \{V, S\}$ , where  $V = Voters$ ,  $S = Shirkers$ ,  $I = Eligible Voter$
- $J \in \{W, L\}$   $J = Outcome$ ,  $W = Win$ ,  $L = Loss$
- $\pi_{i,t}(I, J)$ , payoff at  $t = time\ step$ , for agent  $i$ , (Normal Form Payoff + shock,  
)  $\theta_{i,t}$
- $b_i - c_i$  payoff if  $i$  voted for winning side;  $b_i$  payoff for shirker on winning side
- $-c_i$  for losing voters and 0 for losing shirkers

- $p_{i,t}(V) \in [0, 1]$ , Propensity to Vote
- $a_{i,t}$ , aspirations
- $\varepsilon_p$ , will not adjust propensity
- $\varepsilon_a$ ; will not adjust aspirations

## Propensities

(P1) (positive feedback). For all  $i, t$ , and action  $I \in \{S, V\}$  chosen by  $i$  in  $t$ :

- if  $\pi_{i,t} \geq a_{i,t}$ , then  $\Pr(p_{i,t+1}(I) \geq p_{i,t}(I)) = 1$ ;
- if  $\pi_{i,t} > a_{i,t}$  and  $p_{i,t}(I) < p_i^{\max}$ , then  $\Pr(p_{i,t+1}(I) > p_{i,t}(I)) = 1$ .

(P2) (negative feedback). For all  $i, t$ , and action  $I$  chosen by  $i$  in  $t$ :

- if  $\pi_{i,t} < a_{i,t}$ , then  $\Pr(p_{i,t+1}(I) \leq p_{i,t}(I)) = 1$ ;
- if  $\pi_{i,t} < a_{i,t}$  and  $p_{i,t}(I) > p_i^{\min}$ , then also  $\Pr(p_{i,t+1}(I) < p_{i,t}(I)) = 1$ .

## Aspirations

(A1) For all  $i, t$ :

- if  $\pi_{i,t} > a_{i,t}$ , then  $\Pr(\pi_{i,t} \geq a_{i,t+1} > a_{i,t}) = 1$ .

(A2) For all  $i, t$ :

- if  $\pi_{i,t} = a_{i,t}$ , then  $\Pr(a_{i,t+1} = a_{i,t}) = 1$ .

(A3) For all  $i, t$ :

- if  $\pi_{i,t} < a_{i,t}$ , then  $\Pr(\pi_{i,t} \leq a_{i,t+1} < a_{i,t}) = 1$ .

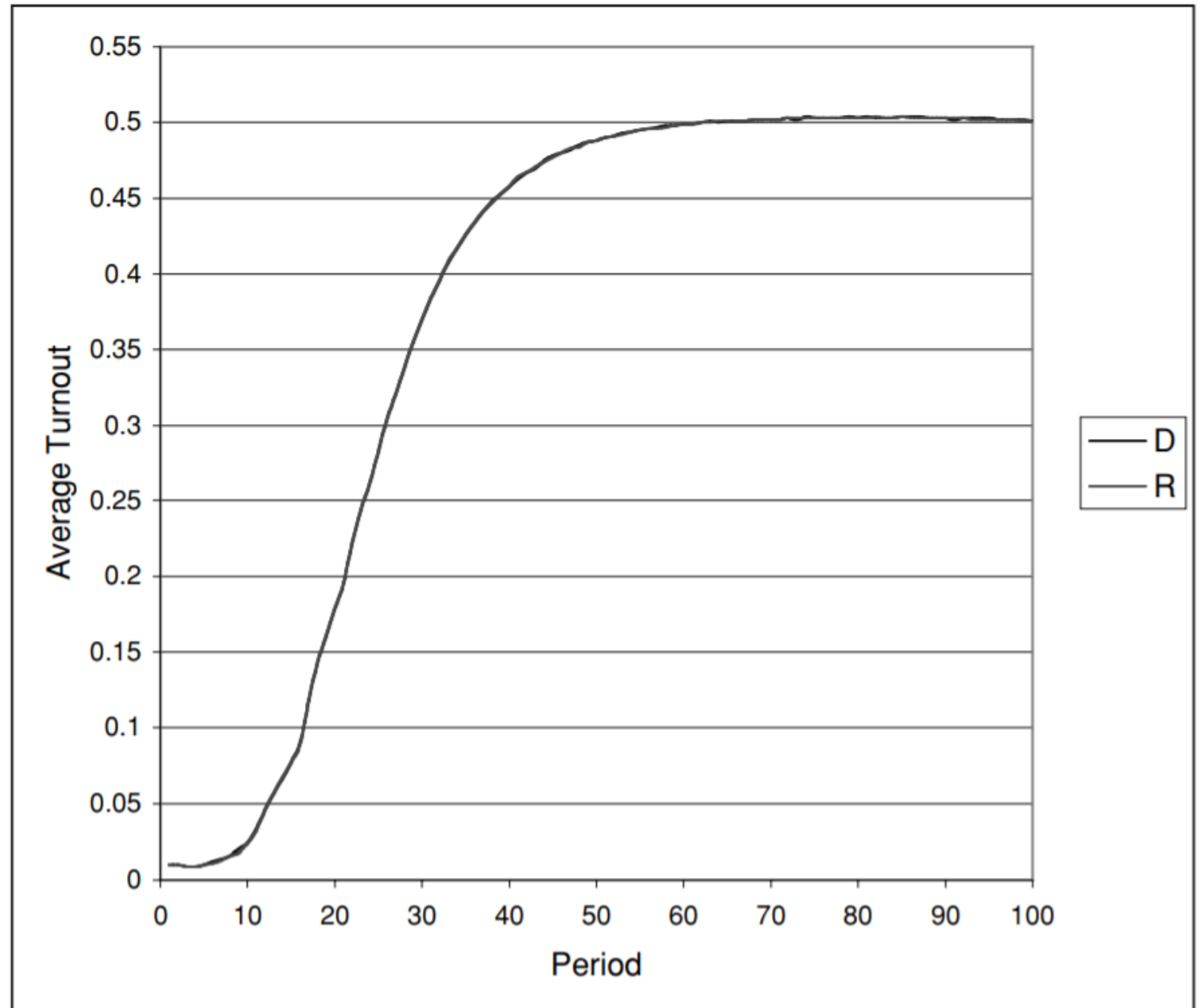
- $p_{i,t+1}(I) = p_{i,t}(I) + \alpha(1 - p_{i,t}(I))$ , Propensity update for winning side
- $p_{i,t+1}(I) = p_{i,t}(I) - \beta p_{i,t}(I)$ , Propensity update for losing side
- $a_{i,t+1} = \lambda a_{i,t} + (1 - \lambda)\pi_{i,t}$ , Aspiration update for winners and losers

# Experiment

- 500,000 Democrats, 500,000 Republicans
- Stabilizes at 50% turnout

Starting Values: 100 Periods  
1,000 Simulations

| <u>Faction</u>           | <u>D</u> | <u>R</u> |
|--------------------------|----------|----------|
| <i>Population</i>        | 5,000    | 5,000    |
| <i>b</i>                 | 1.0      | 1.0      |
| <i>c</i>                 | 0.25     | 0.25     |
| <i>Aspirations</i>       | -0.2     | -0.2     |
| <i>Vote Propensities</i> | 0.01     | 0.01     |





# Altruism and Turnout - James H. Fowler

- Voters will vote if  $PB > C$ ,
  - $P$  = Probability of winning
  - $B$  = Payoff from winning
  - $C$  = Cost of voting
- Incorporate Altruism:  $P(B_s + \alpha NB_o) > C$ .
  - $B_s$  - Payoff for benefit to oneself
  - $B_o$  - Average payoff to rest of the population
  - $\alpha$  – measure of altruism

# Altruism and Turnout: Dictator Game

- Camerer (2003) shows that the mean allocation to player 2 ranges from 10% to 52%.
- $U(\pi_s, \pi_o) = (\pi_s^\rho + \alpha\pi_o^\rho)^{1/\rho}$  , Utility function from dictator game

# Experiment

- 235 subjects were recruited from two introductory undergraduate political science courses
- Subjects were asked whether or not they voted in the March 2004 California primary
- Played the dictator game
- Asked to put themselves along the 7 point scale. 1 being democrat and 7 being Republican

|                                      | Model (1) |      |          |      | Model (2)  |              |            |             |
|--------------------------------------|-----------|------|----------|------|------------|--------------|------------|-------------|
|                                      | Coef.     | S.E. | 95% C.I. |      | Coef.      | S.E.         | 95% C.I.   |             |
| <i>Altruism</i>                      | .5        | (.7) | -.9      | 1.8  | -4.4       | (2.2)        | -8.8       | -.1         |
| <i>Strength of Party ID</i>          | 2.1       | (.7) | .9       | 3.6  | .1         | (1.0)        | -1.8       | 2.3         |
| <b><i>Altruism*Str. Party ID</i></b> |           |      |          |      | <b>6.3</b> | <b>(2.7)</b> | <b>1.0</b> | <b>11.6</b> |
| <i>Constant</i>                      | -3.0      | (.6) | -4.2     | -1.9 | -1.5       | (.8)         | -3.1       | -.1         |

# Future Work

- Improve reinforcement learning based model to get better results
- Formulate voting policies that might encourage voting and evaluate those policies

# Reference

- Economic Theory of Political Action in a Democracy - Anthony Downs (1957)
- The paradox of voter participation? A Laboratory Study - DAVID K. LEVINE and THOMAS R. PALFREY (2005)
- Behavioral Model of Turnout -Jonathan Bendor, Daniel Diermeier, Michael Ting (2003)