## Curling: Why The _ Do You _? <br> Zaheen Ahmad

## Rational Behaviour

- Rational agents play to maximize expected utility in games
- Humans are not always rational in reality
- Difficult to analyze rationality in all games


## Curling



- Sport played on ice
- Two teams, 10 rounds (ends), 16 shots per round


## Curling - Shooting



## Curling - Scoring



## Hammer Shots

- Last shot of an end
- Largely determines the outcome of an end
- Other shots mainly set up the hammer shot
- Teams have a $55.7 \%$ chance of winning beginning game with hammer


## Strategies in Curling

- Intuitively, we'd think about scoring as much as we can per end
- The best sequences of shots to establish a good hammer shot (if we possess it)
- But retain the hammer in ends that count more


## Willoughby and Kostuk, 2004

## Points vs Hammer

- Last end
- Is it better to be:
- +1, without hammer
- -1, with hammer


## Model

$$
P(X=k \mid e, h)
$$

- k, points scored
- e, end number
- h, possession of hammer
- 410 games, 221 up to 10 ends


## Frequency Tables of Scores

| END | -4 | -3 | -2 | -1 | 0 | 1 | 2 | 3 | 4 | 5 |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 10 | 1 | 5 | 4 | 39 | 12 | 113 | 34 | 8 | 4 | 1 | 221 |
| 11 |  |  | 2 | 9 | 4 | 55 | 4 | 1 | 1 |  | 76 |
| 12 |  |  |  |  |  | 3 | 1 |  |  |  |  |

## Results and Comparison

- $E(U P$, Not Hammer) $=0.713$
- $E(D O W N$, Hammer $)=0.287$
- Contrasts with players from survey of 113
- UP, Not Hammer = 41.6
- DOWN, Hammer = 58.4


## Willoughby and Kostuk, 2005

## Blank the 9th End?

- Keep the house clean in 9th end
- TAKE 1 or BLANK end?


## Frequency Tables of Scores

| After 9th | -4 | -3 | -2 | -1 | 0 | 1 | 2 | 3 | 4 | 5 |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0 |  |  | 3 | 15 | 8 | 70 | 12 | 2 |  |  | 110 |
| 1 | 1 | 5 | 4 | 39 | 12 | 113 | 34 | 8 | 4 | 1 | 221 |
| 2 |  | 1 | 1 | 20 | 1 | 16 | 34 | 1 |  |  | 74 |
| 3 |  |  | 1 | 1 | 1 | 1 |  | 1 |  |  | 5 |
|  | 1 | 6 | 9 | 75 | 22 | 200 | 80 | 12 | 4 | 1 | 410 |

## Results of Shots

| Beginning of 9th | E(TAKE) | E(BLANK) |
| :---: | :---: | :---: |
| $\mathbf{3}$ | 1.0000 | 1.0000 |
| $\mathbf{2}$ | 0.9678 | 0.9843 |
| $\mathbf{1}$ | 0.9125 | 0.9263 |
| $\mathbf{0}$ | 0.7050 | 0.8247 |
| $\mathbf{- 1}$ | 0.1753 | 0.2950 |
| $\mathbf{- 2}$ | 0.0737 | 0.0875 |
| $\mathbf{- 3}$ | 0.0157 | 0.0322 |

## Blank the 9th End

- Regardless of situation
- BLANK in 9th end, retain hammer
- Only consider draw for one


## Something's Not Right

- Aggregated -1 and 1 differentials together
- Playing when down by 1 is different than when up by 1
- Only looks at differentials of 1


## Clement, 2012

## Blanking Other Ends

- The author expanded on BLANK or TAKE on other ends
- Multinomial logistic regression + transition matrices


## Regression Model

- Trained on game data
- Features: skill difference, point difference, end number
- Label: the distribution of scores of the end


## Inference

- Sample from the regression model to get distributions at ends
- Create the transition matrix using distributions
- Calculate the win probabilities using the transitions matrix given the scores at each end
- Difference between blanking and taking one (with leads -1 and 1)


## Win Probability Differences



## Win Probability Differences



## Win Probabilities

|  | lead $=-1$ |  |  |  | lead = 1 |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| end | men |  | women |  | men |  | women |  |
|  | blank | take 1 | blank | take 1 | blank | take 1 | blank | take 1 |
| 3 | 0.44 | 0.40 | 0.44 | 0.42 | 0.75 | 0.72 | 0.71 | 0.69 |
| 4 | 0.44 | 0.40 | 0.43 | 0.42 | 0.76 | 0.73 | 0.72 | 0.70 |
| 5 | 0.44 | 0.39 | 0.44 | 0.40 | 0.78 | 0.75 | 0.73 | 0.71 |
| 6 | 0.42 | 0.39 | 0.41 | 0.40 | 0.80 | 0.77 | 0.75 | 0.73 |
| 7 | 0.43 | 0.36 | 0.42 | 0.40 | 0.84 | 0.77 | 0.75 | 0.74 |
| 8 | 0.38 | 0.36 | 0.38 | 0.41 | 0.86 | 0.83 | 0.82 | 0.77 |
| 9 | 0.44 | 0.22 | 0.49 | 0.35 | 0.91 | 0.86 | 0.83 | 0.83 |

- All work only consider differences of 1 point
- Focus on late ends (or aggregates early ends)
- Is it better to blank earlier ends or take points
- Expand to taking more than 1 point


## Win Probability Table

## Lead

|  | $\mathbf{1 0}$ | $\mathbf{9}$ | $\mathbf{8}$ | $\mathbf{7}$ | $\mathbf{6}$ | $\mathbf{5}$ | $\mathbf{4}$ | $\mathbf{3}$ | $\mathbf{2}$ | $\mathbf{1}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathbf{- 4 :}$ | 10.1 | 9.6 | 8.8 | 8.0 | 6.6 | 6.0 | 4.3 | 2.9 | 1.2 | 0.1 |
| $\mathbf{- 3 :}$ | 17.4 | 15.6 | 15.9 | 15.0 | 14.6 | 12.7 | 10.8 | 8.4 | 5.3 | 2.0 |
| $\mathbf{- 2 :}$ | 28.7 | 27.3 | 27.5 | 26.9 | 25.5 | 25.2 | 22.2 | 22.0 | 15.2 | 12.1 |
| -1: | 42.7 | 41.9 | 42.1 | 41.1 | 40.3 | 41.6 | 38.4 | 41.9 | 31.8 | 42.7 |
| +0: | 55.7 | 55.1 | 55.7 | 56.6 | 57.3 | 59.6 | 58.1 | 62.2 | 57.6 | 71.9 |
| $\mathbf{+ 1 :}$ | 71.3 | 70.9 | 72.1 | 72.4 | 74.0 | 75.1 | 75.9 | 79.0 | 83.0 | 88.4 |
| $\mathbf{+ 2 :}$ | 81.8 | 83.2 | 82.8 | 84.8 | 85.5 | 86.9 | 88.3 | 91.3 | 94.3 | 98.0 |
| $\mathbf{+ 3 :}$ | 89.9 | 90.2 | 91.1 | 91.9 | 93.0 | 93.8 | 95.3 | 97.3 | 98.6 | 99.7 |
| $\mathbf{+ 4 :}$ | 94.9 | 95.2 | 95.8 | 96.3 | 97.3 | 97.6 | 98.7 | 99.2 | 99.7 | 100.0 |

## Approaches

- More complex models to learn better representations of data
- Simulated experiments
- Curling simulator
- Al search for strategies and outcomes

