Mechanism Design

CMPUT 654: Modelling Human Strategic Behaviour

S&LB §10.1-10.2

Logistics

- **Assignment #2** will be released on **Thursday**
- See the course schedule for paper presentation assignments
- Assignment #1 is about half-marked; should have results by the end of the week
- I will email solutions to Assignment #1 when it is marked; please **do not share the solutions** with anyone outside the class

Recap: Social Choice

Definition: A **social choice function** is a function $C : L^n \to O$, where

- $N = \{1,2,\ldots,n\}$ is a set of **agents**
- O is a finite set of **outcomes**
- L is the set of (non-strict) **total orderings** over O.

Definition: A **social welfare function** is a function $C : L^n \to L$, where N, O, and L are as above.

Notation:
We will denote **i's preference order** as $\succeq_i \in L$, and a **profile** of preference orders as $[\succeq] \in L^n$.

Recap: Voting Scheme Properties

Definition:
W is **Pareto efficient** if for any $o_1, o_2 \in O$,

$$(\forall i \in N : o_1 \succ_i o_2) \implies (o_1 \succ_W o_2).$$

Definition:
W is **independent of irrelevant alternatives** if, for any $o_1, o_2 \in O$ and any two preference profiles $[\succ'], [\succ''] \in L$,

$$(\forall i \in N : o_1 \succ'_i o_2 \iff o_1 \succ''_i o_2) \implies (o_1 \succ_{W[\succ']} o_2 \iff o_1 \succ_{W[\succ'']} o_2).$$

Definition:
W does not have a **dictator** if

$$\neg i \in N : \forall [\succ] \in L^n : \forall o_1, o_2 \in O : (o_1 \succ_i o_2) \implies (o_1 \succ_W o_2).$$

Recap: Arrow's Theorem

Theorem: (Arrow, 1951)
If $|O| > 2$, any social welfare function that is Pareto efficient and independent of irrelevant alternatives is dictatorial.

- Unfortunately, restricting to social choice functions instead of full social welfare functions doesn't help.

Theorem: (Muller-Satterthwaite, 1977)
If $|O| > 2$, any social choice function that is weakly Pareto efficient and monotonic is dictatorial.

Lecture Outline

1. Recap & Logistics
2. Mechanism Design with Unrestricted Preferences
3. Quasilinear Preferences

Mechanism Design

- In the social choice lecture, we assumed that agents report their preferences **truthfully**
- We now allow agents to report their preferences **strategically**
- Which social choice functions are **implementable** in this new setting?
    - **Question:** Wait, didn't we prove that social choice was hopeless?

Bayesian Game Setting

Definition:
A **Bayesian game setting** is a tuple (N, O, Θ, p, u) where

- N is a finite set of n **agents**,
- O is a set of **outcomes**,
- $\Theta = \Theta_1 \times \cdots \times \Theta_n$ is a set of possible **type profiles**,
- p is a **common prior** distribution over Θ, and
- $u = (u_1, \ldots, u_n)$, where $u_i : O \to \mathbb{R}$ is the **utility function** for player i.

This differs from a Bayesian game only in that utilities are defined on **outcomes** rather than **actions**, and agents are not (yet) endowed with an action set.

Mechanism

Definition:

A **mechanism** for a Bayesian game setting (N, O, Θ, p, u) is a pair (A, M), where

- $A = A_1 \times \cdots A_n$, where A_i is the set of **actions** available to agent i, and
- $M : A \rightarrow \Delta(O)$ maps each **action profile** to a distribution over **outcomes**

Intuitively, a **mechanism designer** (sometimes called **The Center**) needs to decide among outcomes in some Bayesian game setting, and so they design a mechanism that **implements** some social choice function.

Dominant Strategy Implementation

Definition:
Given a Bayesian game setting (N, O, Θ, p, u), a mechanism (A, M) is an **implementation in dominant strategies** of a social choice function C (over N and O) if,

1. The Bayesian game $(N, A, \Theta, p, u \circ M)$ induced by (A, M) has an equilibrium in dominant strategies, and

2. In any such equilibrium s^*, and for any type profile $\theta \in \Theta$, we have $M(s^*(\theta)) = C(u(\cdot, \theta))$.

Bayes-Nash Implementation

Definition:
Given a Bayesian game setting (N, O, Θ, p, u), a mechanism (A, M) is an **implementation in Bayes-Nash equilibrium** of a social choice function C (over N and O) if

1. There exists a Bayes-Nash equilibrium of the Bayesian game $(N, A, \Theta, p, u \circ M)$ induced by (A, M) such that

2. for every type profile $\theta \in \Theta$ and action profile $a \in A$ that can arise in equilibrium, $M(a) = C(u(\cdot, \theta))$.

The Space of All Mechanisms Is Enormous

- The space of all functions that map actions to outcomes is **impossibly large** to reason about
- **Question:** How could we ever prove that a given social choice function is **not implementable**?
- Fortunately, we can restrict ourselves without loss of generality to the class of **truthful, direct** mechanisms

Direct Mechanisms

Definition: A **direct** mechanism is one in which $A_i = \Theta_i$ for all agents $i \in N$.

Definition:
A direct mechanism is **truthful** (or **incentive compatible**) if, for all type profiles $\theta \in \Theta$, it is a dominant strategy in the game induced by the mechanism for each agent to report their true type.

Definition:
A direct mechanism is **Bayes-Nash incentive compatible** if there exists a Bayes-Nash equilibrium of the induced game in which every agent always truthfully reports their type.

Revelation Principle

Theorem: (Revelation Principle)
If there exists **any** mechanism that implements a social choice function C in dominant strategies, then there exists a **direct** mechanism that implements C in dominant strategies and is **truthful**.

- Identical result for implementation in Bayes-Nash equilibrium

Revelation Principle Proof

1. Let (A, M) be an **arbitrary mechanism** that implements C in Bayesian game setting (N, O, Θ, p, u).

2. Construct the **revelation mechanism** $(\Theta, \overline{M})$ as follows:

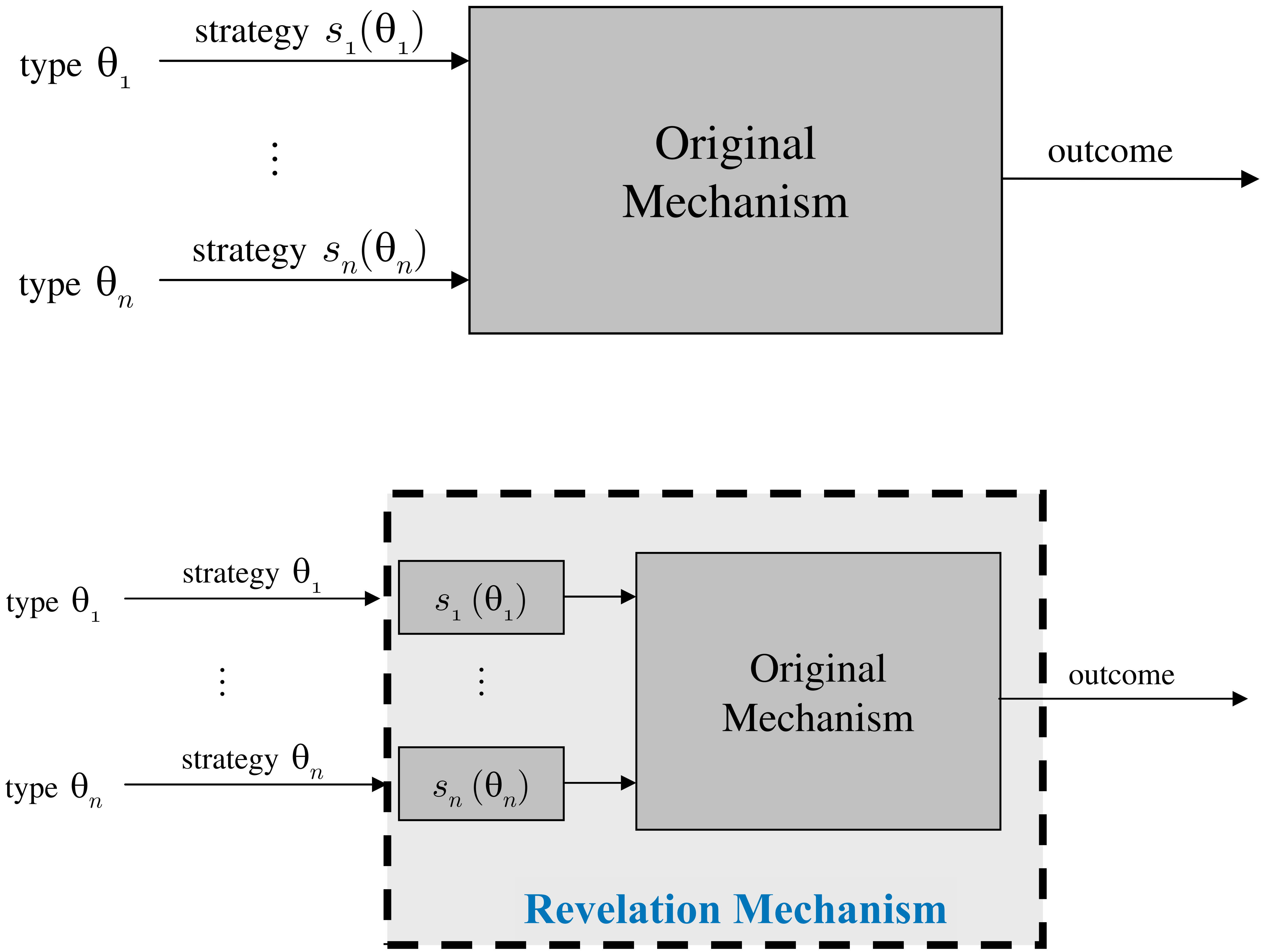
   - For each type profile $\theta \in \Theta$, let $a^*(\theta)$ be the action profile in which every agent plays their dominant strategy in the game induced by (A, M).

   - Define $\overline{M}(\theta) = M(a^*(\theta))$.

3. Each agent reporting type $\hat{\theta}_i$ will yield the same outcome as every agent of type $\hat{\theta}_i$ playing their dominant strategy in M

4. So it is a dominant strategy for each agent to report their true type $\hat{\theta}_i = \theta_i$.

Revelation Mechanism



(Image: Shoham & Leyton-Brown 2008)

General Dominant-Strategy Implementation

Theorem: (Gibbard-Satterthwaite)
Consider any social choice function C over N and O. If $|O| > 2$ (there are at least **three** outcomes),

1. C is **onto**; that is, for every outcome $o \in O$ there is a preference profile $[\succ]$ such that $C([\succ]) = o$ (this is sometimes called **citizen sovereignty**), and

2. C is dominant-strategy **truthful**,

then C is **dictatorial**.

Hold On A Second

Haven't we already seen an example of a dominant-strategy truthful direct mechanism?

Second Price Auction

- **Outcomes** are $O = \{(i \text{ gets object, pays } \$x) \mid i \in N, x \in \mathbb{R}\}$
- **Types** are $\theta_i = \mathbb{R}$, where an agent i with type $x \in \mathbb{R}$ has preferences:

  $(i \text{ gets object, pays } \$y') \succ_i (i \text{ gets object, pays } \$y'')$ for all $y' < y''$ and $y' < x$,

  $(i \text{ gets object, pays } \$y') \succ_i (j \text{ gets object, pays } \$y'')$ for all $y' < x$ and $i \neq j$,

  $(j \text{ gets object, pays } \$y'') \succ_i (i \text{ gets object, pays } \$y')$ for all $y' > x$ and $i \neq j$.
- **Social choice function**: Assign the item to the agent with the highest type
- **Actions**: Agents directly announce their type via sealed bid

- **Question:** Why is this not ruled out by Gibbard-Satterthwaite?

Restricted Preferences

- Gibbard-Satterthwaite only applies to social choice functions that operate on **every possible** preference ordering over the outcomes
- By **restricting the set of preferences** that we operate over, we can circumvent Gibbard-Satterthwaite

Quasilinear Preferences

Definition:
Agents have **quasilinear preferences** in an n-player Bayesian game setting when

1. the set of outcomes is $O = X \times \mathbb{R}^n$ for a finite set X,
2. the utility of agent i given type profile θ for an element $(x, p) \in O$ is $u_i\big((x,p),\theta\big) = v_i(x,\theta) - f_i(p_i)$, where
3. $v_i : X \times \Theta \to \mathbb{R}$ is an **arbitrary** function, and
4. $f_i : \mathbb{R} \to \mathbb{R}$ is a **monotonically increasing** function.

Quasilinear Preferences, informally

- **Intuitively:** Agents' preferences are split into
  1. finite set of **nonmonetary** outcomes (e.g., allocation of an object)
  2. monetary **payment** made to **The Center** (possibly negative)
- These two preferences are **linearly** related
- Agents are permitted **arbitrary preferences** over nonmonetary outcomes, but **not over payments**
- Agents care only about the **outcome selected** and their **own payment**
  - *and*, the amount they care about the outcome is **independent** of their payment

Direct Quasilinear Mechanism

Definition:

A **direct quasilinear mechanism** is a pair (χ, p), where

- $\chi : \Theta \to \Delta(X)$ is the **choice rule** (often called the **allocation rule**), which maps from a profile of reported types to a distribution over nonmonetary outcomes, and

- $p : \Theta \to \mathbb{R}^n$ is the **payment rule**, which maps from a profile of reported types to a payment for each agent.

Summary

- **Mechanism design**: Setting up a system for **strategic agents** to provide input to a **social choice function**
- **Revelation Principle** means we can restrict ourselves to **truthful direct** mechanisms without loss of generality
- Non-dictatorial dominant-strategy mechanism design is **impossible in general** (Gibbard-Satterthwaite)
- The special case of **quasi-linear preferences** will allow us to **circumvent** Gibbard-Satterthwaite (next time!)