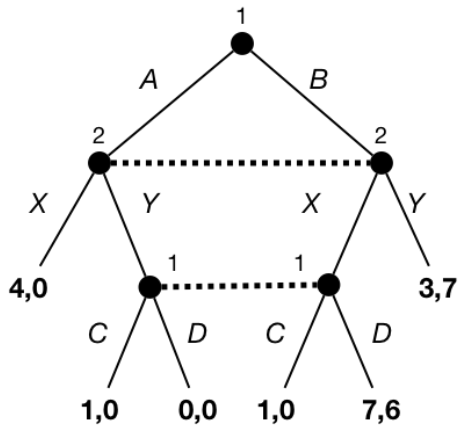
CMPUT 654, Fall 2019
Assignment #2

Due: Thursday, Oct. 24, 2019, 1:59pm
Total points: 92

Name:
Student number:
Collaborators and resources:

1. **(Imperfect Information Extensive Form Games)**
   Consider the following imperfect information extensive form game.



   (a) **[5 points]** List the pure strategies for each agent.

   (b) **[5 points]** Find two pure strategy Nash equilibria of the game. Justify your answer.

   (c) **[5 points]** Is this a game of perfect recall, or imperfect recall? Justify your answer.

2. **(Repeated Games)**

Consider the following stage game:

	X	Y	Z
X	$10, 10$	$8, 11$	$0, 20$
Y	$11, 8$	$9, 9$	$7, 10$
Z	$5, 0$	$10, 7$	$8, 8$

G_{stage}

(a) **[5 points]** Suppose that G_{stage} will be repeated 3 times. Give a *fully specified* pure strategy Nash equilibrium of the repeated game.

(b) **[5 points]** Now suppose that G_{stage} will be repeated infinitely many times, and the overall payoff from the repeated game is the limit of average payoffs. Consider the following pure strategy profile: Row player plays X every game; column player plays X on even rounds and Z on odd rounds. If either player deviates from this plan, the other player will play Z forever.

i. What is the payoff to each player of this strategy profile?

ii. Is this strategy profile a Nash equilibrium? Why or why not?

(c) **[5 points]** Suppose that G_{stage} will be repeated infinitely many times, and the overall payoff from the repeated game is the limit of average payoffs. Consider the following pure strategy profile: Row player plays X every game unconditionally; column player plays X on even rounds and Y on odd rounds, also unconditionally.

i. What is the payoff to each player of this strategy profile?

ii. Is this strategy profile a Nash equilibrium? Why or why not?

(d) **[10 points]** Suppose that G_{stage} will be repeated infinitely many times, and the overall payoff from the repeated game is the limit of average payoffs. Construct a pure strategy equilibrium in which both agents receive a payoff of 10.

3. **(Bayesian Games)**

Roy and Colleen are each considering whether to go to the office. They've never worked together, and so neither knows whether the other is a quiet or a loud officemate. However, they both know that their loudness is drawn from a commonly known joint distribution:

Roy	Colleen	p
loud	loud	.2
loud	quiet	.5
quiet	loud	.1
quiet	quiet	.2

Working in the office alone is very productive, and yields 40 units of utility. Working in the office with a quiet officemate is just as productive, and yields 40 utility also. Working in the office with a loud officemate when you yourself are loud is not as good as working alone, and only yields utility 20. Working in the office with a loud officemate when you yourself are quiet is intolerable, and yields -30 utility. Staying at home instead of going to the office is unproductive and yields utility 0.

(a) **[5 points]** Represent this scenario as a Bayesian game. You will need to specify a set of agents N, a set of actions A_i for each agent, a set of types Θ_i for each agent, a probability function $p_i : \Theta_i \to \Delta(\Theta_{-i})$ mapping from each agent's type to a distribution over the types of the other agents, and a utility function $u_i : A \times \Theta \to \mathbb{R}$ mapping from action profile and type profile to a utility for each agent.

Let $N = \{r, c\}$ be the set of agents, $\Theta = \Theta_r \times \Theta_c$ be the set of type profiles, and let O represent the action of going to the office and H represent the action of staying home.

(b) **[2 points]** What is the *ex-ante* expected utility to Roy of the strategy profile (OH, OO), where OH is the pure strategy that maps type *loud* to action O and maps type *quiet* to action H?

(c) **[2 points]** What is the *ex-interim* expected utility to Roy of the strategy profile (OH, OO) when his type is `loud`?

(d) **[2 points]** What is the *ex-post* expected utility to Roy of the strategy profile (OH, OO) when his type is `loud` and Colleen's type is also `loud`?

(e) **[8 points]** Draw the payoff matrix of the induced normal form for this game. Explicitly state what each pure strategy in the normal form game means. Please list the pure strategies in alphabetical order.

(f) **[5 points]** Find all of the Bayes-Nash equilibria of this game. Justify your response.

4. **(Mechanism Design)**

(a) **[15 points]**
Construct a direct mechanism for the scenario in question (3) that implements maximization of social welfare (i.e., sum of the agents' utilities) in Bayes-Nash equilibrium, and is Bayes-Nash incentive compatible.

(b) **[5 points]** Does the mechanism you constructed in question (4a) have an *ex-post* equilibrium? Why or why not?

5. **(Social Choice)**

   A group of $n > 2$ agents live along a straight road, each at an integer number of miles from the end of the road. They want to aggregate their preferences over the location of a new library, which will also be placed on an integer mile. Each agent has *single-peaked preferences* over locations: they prefer locations that are closer to their home to all further locations. (Note that these are non-strict preferences, since an agent who lives at mile 5 will be indifferent between putting the library at mile 3 or mile 7.)

   They decide to use the following social welfare function W to rank candidate locations: Each agent reports the location that they most prefer (i.e., their home), and the aggregated preferences are induced by distance from the average of those locations. So if the average of the locations is 7, then $7 \succ_W 6 \succeq_W 8 \succ_W 5 \ldots$.

   (a) **[2 points]** Is W Pareto efficient? Why or why not?

   (b) **[2 points]** Is W dictatorial? Why or why not?

   (c) **[4 points]** Is W independent of irrelevant alternatives? Why or why not?

Submission

Each assignment consists of a *problem set* file in PDF format containing the list of questions to answer. Each assignment is to be submitted electronically via GradeScope before the due date. The submission should consist of a PDF file containing the answers to the problem set.

A LaTeX file is provided that you can edit to produce a PDF of your answers if you wish. Otherwise, you can type your answers into your favourite word processor and print to PDF, or you can write your answers (legibly!) by hand and upload a scan.