CMPUT 654, Fall 2019
Assignment #1

Due: Tuesday, Oct. 1, 2019, 1:59pm
Total points: 155

Name:
Student number:
Collaborators and resources:

1. **(Utility Theory)**

   (a) **[20 points]** Prove that it is not always possible to represent preferences by maximization of expected utility if the preferences do not satisfy von Neumann and Morgenstern's *Decomposability* condition. Hint: construct a preference order that satisfies all of the other conditions, but does not satisfy *Decomposability*, and show that this cannot be represented by expected utility maximization.

   (b) **[5 points]** Consider the following game: A fair coin is tossed. If it comes up heads, you win \$2. Otherwise, the coin is flipped again. If it comes up heads on the second flip, you win \$4. Otherwise, the coin is flipped again, and so on. That is, the coin is flipped until the first heads appears on the kth flip; and then you are paid 2^k dollars.

   Prove that the expected monetary value of this game is infinite.

(c) **[15 points]** Suppose that a perfectly rational expected utility maximizer is given the opportunity to play the game from question (1b). Suppose further that the agent has strictly monotonic value for money: $\$(x+1) \succ \$x$ for all values of x. Is there any value of y such that this agent can rationally turn down the opportunity to play the game in exchange for $\$y$? (In other words, do there exist and $y \in \mathbb{R}$ and rational preferences $\succeq$ for which $\$y \succ$ (game from question 1b)).

Either construct a preference order $\succeq$ that satisfies all of the von Neumann-Morgenstern axioms, strictly monotonic value for money, and $\$y \succ$ (game from question 1b), or prove that no such preference order exists.

2. **(Normal Form Games)**

   Consider the following game:

	L	M	R	Q
W	$3, 0$	$-9, 4$	$-1, 1$	$4, 0$
X	$9, 1$	$2, -1$	$3, 1$	$5, 3$
Y	$-2, 7$	$1, 0$	$2, 3$	$3, 7$
Z	$-4, 4$	$12, 5$	$4, 6$	$2, 4$

G

   (a) [**2 points**] List the Pareto optimal outcomes in game G.

   (b) [**3 points**] List the weakly dominated pure strategies in game G.

   (c) [**2 points**] List the strictly dominated pure strategies in game G.

   (d) [**3 points**] Which pure strategies survive iterated removal of strictly dominated strategies in game G? Justify your response.

   (e) [**10 points**] For each rationalizable pure strategy of G, provide a rationalizable belief to which the strategy is a best response. For each unrationalizable pure strategy of G, explain why the strategy is not rationalizable.

   (f) [**10 points**] Compute and list all of the Nash equilibria in game G. Justify your responses.

3. **(More Normal Form Games)**

   (a) **[5 points]** Construct a normal-form game in which an agent plays a strictly dominated action in some Nash equilibrium and provide the equilibrium, or prove that this is impossible.

   (b) **[5 points]** Construct a normal-form game in which an agent plays a weakly dominated action in some Nash equilibrium and provide the equilibrium, or prove that this is impossible.

   (c) **[5 points]** Construct a normal-form game in which an agent plays a very weakly dominated action in some Nash equilibrium and provide the equilibrium, or prove that this is impossible.

   (d) **[10 points]** Consider the following pair of utility functions for money outcomes:

$$\begin{aligned} u_1(\$0) &= 0 & \qquad u_2(\$0) &= 0 \\ u_1(\$1) &= 1 & \qquad u_2(\$1) &= 1 \\ u_1(\$3) &= 3 & \qquad u_2(\$3) &= 1.75 \\ u_1(\$6) &= 6 & \qquad u_2(\$6) &= 3 \end{aligned}$$

   Let $\ell = [.5 : \$6, .5 : \$0]$ be a lottery that pays out \$6 and \$0 with equal probability.

   Consider the following game: Players 1 and 2 have the utilities specified above and identical net worths of \$0. They simultaneously choose between actions X and Y. If both players choose X, then player 1 receives the outcome of lottery ℓ and player 2 receives \$6. If both players choose Y, then player 1 receives \$1 and player 2 receives the outcome of lottery ℓ. If one player chooses X and the other chooses Y, then the player who chose X receives \$0 and the player who chose Y receives \$1.

   Represent this situation as a **normal form game** and list all of its Nash equilibria.

4. **(Maxmin and Minmax)**

   (a) **[5 points]** What is the row player's maxmin value in game G from question 2? Justify your response.

   (b) **[5 points]** What is the column player's minmax value in game G from question 2? Justify your response.

   (c) **[5 points]** Prove that a player's maxmin value is less than or equal to their minmax value in all games.

   (d) **[5 points]** Prove that a player's maxmin value is equal to their minmax value in all two-player games.

5. **(Correlated Equilibrium)**

(a) **[10 points]** Consider the following game:

	L	C	R
T	$2, -2$	$7, -7$	$-11, 11$
M	$8, -8$	$10, -10$	$15, -15$
B	$-10, 10$	$-3, 3$	$-1, 1$

G_1

Construct a correlated equilibrium for game G_1 in which the column player receives an expected utility of 4, or prove that no such correlated equilibrium exists.

(b) **[10 points]** Consider the following game:

	L	C	D	R
T	$20, 4$	$16, 8$	$12, 5$	$3, 3$
M	$14, 13$	$0, 0$	$-2, -2$	$5, 12$
B	$13, 13$	$7, 7$	$0, 0$	$8, 16$
E	$5, 5$	$13, 13$	$13, 14$	$4, 20$

G_2

Construct a correlated equilibrium for game G_2 in which the row player receives an expected utility of 14 and the column player receives an expected utility of 10, or prove that no such correlated equilibrium exists.

(c) **[10 points]** Construct a correlated equilibrium for game G_2 in which both players play only two actions with positive probability and receive an expected utility of 13.5, or prove that no such correlated equilibrium exists.

(d) **[10 points]** Consider the following game:

	L	C	R
T	$5, 10$	$16, 8$	$0, 0$
M	$0, -8$	$0, 0$	$8, 16$
B	$6, 0$	$-8, 0$	$10, 5$

G_3

Let $v = (v_1, v_2)$ be signals over the actions for the row and column player respectively. Also let $\sigma = (\sigma_1, \sigma_2)$ be the identity mapping; that is, the signals recommend the actions for players to take. Together with following distribution π over signals, is (v, π, σ) a correlated equilibrium? Justify your answer.

$$\pi(v_1, v_2) = \begin{cases} \frac{1}{6} & \text{if } v_1 = T, v_2 = C, \\ \frac{1}{12} & \text{if } v_1 = T, v_2 = R, \\ \frac{1}{12} & \text{if } v_1 = M, v_2 = C, \\ \frac{1}{6} & \text{if } v_1 = M, v_2 = R, \\ \frac{1}{2} & \text{if } v_1 = B, v_2 = R, \\ 0 & \text{otherwise.} \end{cases}$$

Submission

Each assignment consists of a *problem set* file in PDF format containing the list of questions to answer. Each assignment is to be submitted electronically via GradeScope before the due date. The submission should consist of a PDF file containing the answers to the problem set.

A LaTeX file is provided that you can edit to produce a PDF of your answers if you wish. Otherwise, you can type your answers into your favourite word processor and print to PDF, or you can write your answers (legibly!) by hand and upload a scan.