Local Search

CMPUT 261: Introduction to Artificial Intelligence

P&M §4.7

Logistics & Assignment #1

- **Assignment #1 was released last week**
  See eClass
  - Due **January 31** at 11:59pm
- Office hours have begun!
  - Not mandatory; for getting help from TAs
  - **There are no labs for this course:** You do not need to show up for your scheduled lab section
- There will be an **example/practice midterm**

Recap

- **Search problems** are an extremely general encoding for choosing a **sequence of actions** from a start state to a goal state
- Using **heuristic functions** can speed this process up
  - **A* search** is optimal but space-intensive
  - **Branch & bound depth-first search** is optimal and space efficient, but needs a good starting bound
  - **Iterative Deepening A* (IDA*)** finds a good bound by iterative restarts (like IDS), but can be quadratically less time-efficient
- Varying the direction of search can exploit mismatches in forward and reverse **branching factors**

Lecture Outline

1. Recap & Logistics
2. Local Search
3. Hill Climbing
4. Randomized Algorithms

After this lecture, you should be able to:

- Implement stochastic local search and demonstrate its operation
- Implement simulated annealing and demonstrate its operation
- Identify when stochastic local search is more appropriate than graph search
- Explain the relative advantages and disadvantages of different neighbourhood specifications

Searching for Goal Nodes

Sometimes, we know how to **recognize** a goal node, but not how to **construct** one.

Example (SAT problem): Given a Boolean formula,

$$P(X) = (X_1 \vee X_2 \vee \neg X_3) \wedge \ldots \wedge (\neg X_{k-2} \vee \neg X_{k-1} \vee X_k),$$

is there an assignment of truth values to the variables X_i that makes the formula true?

- **State** is the values of the different variables
- **Easy to recognize** when we've succeeded, but computing a "satisfying assignment" is **NP-complete** in general
- **SAT** is an example of a **constraint satisfaction problem**

Searching for Goal Nodes

We can encode SAT as a graph search problem (assignments as states, variable value changes as actions), *but*:

1. The **space is too big** to explore exhaustively
    - **Question:** How many states are there in a SAT problem with k variables?
    - Industrial SAT problems routinely have **hundreds of thousands** of variables
2. We don't care about the **sequence of actions**
    - Once we have a satisfying assignment, we are done
    - In fact, there isn't even a "real" set of actions; we have to make something up!

Local Search

- **Idea:** start from a random assignment, and then search around in the space of possible assignments
- Need not keep track of the sequence of moves that we took
- Intuitively:
    1. Select an assignment of a value to each variable
    2. Repeat:
        (i) Select a variable to change
        (ii) Select a new value for that variable
    3. until a satisfying assignment is found

Local Search Problem

Definition: Local Search Problem

- A **constraint satisfaction problem**: A set of **variables**, **domains** for the variables, and **constraints** on their joint assignment.
- **Neighbours function**: $neighbours(n)$
  - Maps from a node n to a set of "similar" nodes
- **Score function**: $score(n)$
  - Evaluates the "quality" of an assignment

Questions:

1. What are the **nodes**?
2. What are the **goal nodes**?

Neighbourhoods

- In previous graph search problems, the **successor function** represents states that can be **reached** from a given state by taking some actual action
    - In local search problems, the **neighbours function** is a **design decision**
    - We choose actions that will help us efficiently **explore the space** rather than trying to represent **actual actions**
- Usually the **neighbourhood** is states that differ in **small ways** from the current state (variable assignment)
    - E.g.: Assignments that differ in k different variables, possibly by a small amount
- **Question:** What might be a good **neighbourhood** function for **SAT**?

Heuristics vs. Scores

- Previously, the heuristic was **optional**, for improving efficiency
- In local search problems, the **score** function is **required**
    - The state space is **too big** to exhaustively explore, so uninformed search is not an option
    - Sometimes we don't even have a goal, we just want to **maximize the quality** of the state
- Example scores: number of satisfied clauses (in SAT); number of satisfied constraints (in CSP)
    - Note: we maximize a score (**why?**)
    - Unlike graph search, score doesn't need to be positive (**why?**)

Generic Local Search Algorithm

Input: a constraint satisfaction problem; a *neighbours* function;
a *score* function to maximize; a *stop_walk* criterion

current := random assignment of values to variables
incumbent := *current*
repeat
    if *incumbent* is a satisfying assignment:
        return *incumbent*
    if *stop_walk*():
        current := new random assignment of values to variables
    else:
        select a *current* from *neighbours*(*current*)
    if *score*(*current*) > score(incumbent):
        incumbent := current
until termination

Hill Climbing

- **Idea:** Select the neighbour with the **highest score**
  - This is called an **improving step**
  - If no improving steps available, **halt** and return *incumbent*
- We'll move toward the best solution once we are close enough
- This algorithm is called **hill climbing**:
  - It seeks the **highest point** on the scoring function's graph
  - It moves only **uphill** (i.e., it makes only improving steps)

Hill Climbing Algorithm

Input: a constraint satisfaction problem; a *neighbours* function; a *score* function

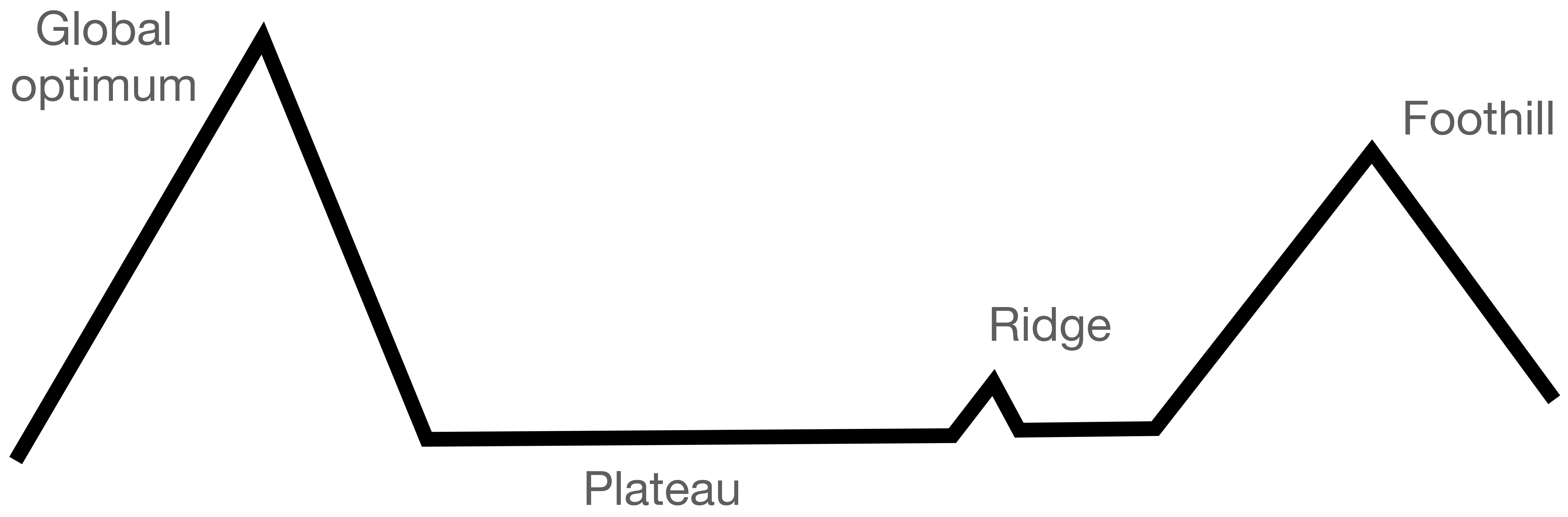
current := random assignment of values to variables
incumbent := *current*
repeat
    if *incumbent* is a satisfying assignment:
        return *incumbent*
    if *False*:
        current := new random assignment of values to variables
    else:
        current := *n* from *neighbours*(*current*) with maximum *score*(*n*)
    if *score*(*current*) > *score*(*incumbent*):
        incumbent := *current*
    else:
        return *incumbent*
until termination

Questions:

1. Is hill climbing **complete**?
2. Is hill climbing **optimal**?

Hill Climbing Problems

1. **Foothills:** Local maxima that are not global maxima
2. **Plateaus:** Regions of the state space where the score is uninformative
3. **Ridges:** Foothills that would not be foothills with a larger neighbourhood
4. **Ignorance of the global optimum:** Unless we reach a satisfying assignment, we cannot be sure that an optimum returned by local search is the global optimum.



Randomized Algorithms

- Adding **random moves** can fix some hill climbing problems
- Two main kinds of random move:
    1. **Random restart:** Start searching from a **completely** random new location
    2. **Random step:** Choose a random **neighbour**
- **Stochastic local search:** Add both kinds of random moves to hill climbing

Stochastic Local Search

Input: a constraint satisfaction problem; a *neighbours* function; a *score* function to maximize; a *stop_walk* criterion; a *random_step* criterion

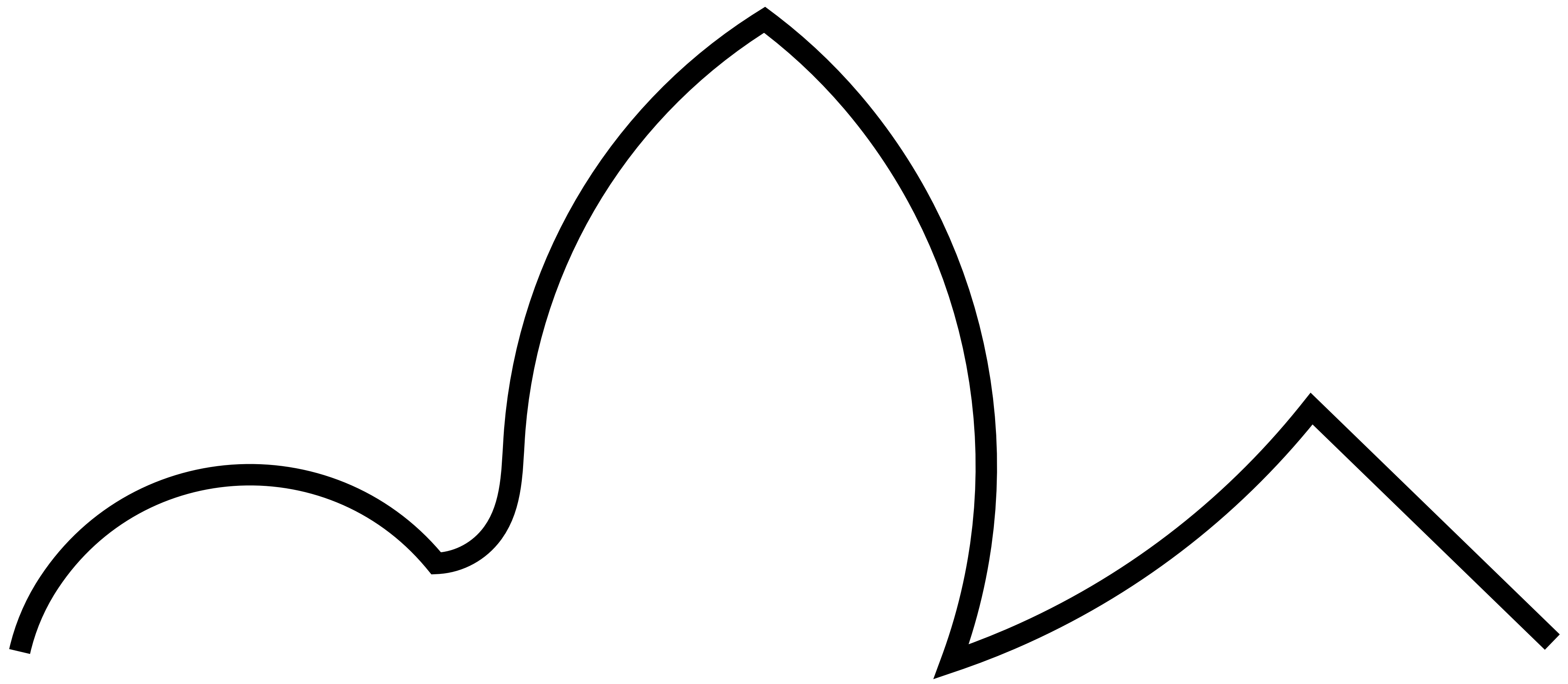
current := random assignment of values to variables
incumbent := *current*
repeat
    if *incumbent* is a satisfying assignment:
        return *incumbent*
    if *stop_walk*():
        current := new random assignment of values to variables
    else if *random_step*():
        current := a random element from *neighbours*(*current*)
    else:
        current := *n* from *neighbours*(*current*) with maximum *score*(*n*)
    if *score*(*current*) > score(incumbent):
        incumbent := current

Questions:

1. Is stochastic local search **complete**? (**Why?**)
2. Is stochastic local search **optimal**? (**Why?**)

Two Examples

- Consider two *partial* algorithms:
  1. Hill climbing plus **random restart only**
  2. Hill climbing plus **random steps only**
- **Question:** Which finds the maximum most easily on each of these two search spaces? Why?



Sampling from a Neighbourhood

- So far we have a sharp distinction between **random steps** and **greedy steps**
- We either
  1. Move to a new current state that **maximizes** the score in the neighbourhood, *OR*
  2. Choose a new current state completely at random from the neighbourhood, **regardless of its score**
- **Question:** Is there something in between that we could do?
- **Simulated annealing:**
  - Choose a random neighbour
  - Make a random decision about whether to "accept" that neighbour that depends on the new neighbour's score

Simulated Annealing

Input: a constraint satisfaction problem; a *neighbours* function; a *score* function to maximize; a "temperature schedule" $T_1, T_2, \ldots$

$current$:= random assignment of values to variables
$incumbent$:= $current$

repeat for $T = T_1, T_2, \ldots$
  Randomly choose new from $neighbours(current)$
  if $score(new) > score(current)$
    always accept (i.e., $current$:= new)
  else
    accept with **probability**

$$e^{[(score(new) - score(current))/T]}$$

Simulated Annealing cont.

$$e^{[\boxed{(score(new)-score(current))}/\boxed{T}]}$$

- Worse $score(new)$ means lower acceptance probability
- Always negative (**why?**)

- Higher T makes negative value smaller
- Higher acceptance probability

- Small neighbourhoods are good, because they are more efficient to search
- Large neighbourhoods are good, because they are more likely to contain an improvement
- Simulated annealing allows for a large neighbourhood *and* efficient searching
  - You don't have to generate the whole neighbourhood for greedy steps, just randomly construct a **single** neighbour
  - But you will tend to move to parts of the state space with good score
- Can also change sampling distribution over time
  - More random in early iterations, more likely to be a "nearby" state in later iterations

Summary

- For some problems, we only care about finding a **goal node**, not the actions we took to find it
- **Local search:** Look for goal states by iteratively moving from a **current state** to a **neighbouring state**
  - **Hill climbing:** Always move to the **highest-score** neighbour
  - **Random step:** Sometimes choose a **random** neighbour
  - **Random restart:** Sometimes start again from an **entirely random** state
  - **Simulated annealing:** Every move is random, but the sampling distribution is increasingly non-uniform, and we don't always "accept" the sampled state