Game Theoretical Approaches to the Handling Road Traffic

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Overview

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    - Users
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What is Traffic?

“Traffic can be defined as the movement of pedestrians and goods along a route, and in the 21st century the biggest problem and challenge for the traffic engineer is often the imbalance between the amount of traffic and the capacity of the route, leading to congestion. Traffic congestion is not a new phenomenon. Roman history records that the streets of Rome were so clogged with traffic, that at least one emperor was forced to issue a proclamation threatening the death penalty to those whose chariots and carts blocked the way. “

Slinn, M., Guest, P., & Matthews, P. (2005). *Traffic engineering design : Principles and practice* (Second ed.).

Motivation

- Maintain safety
- Avoid congestion
- Reduce time and resource waste
- Reduce environmental impact
    - In southern California it was found that CO_2 emission can be reduced by 20% with better traffic control Barth, M., & Boriboonsomsin, K. (2008)

Traffic engineering

- Traffic engineering is an established branch of civil engineering.
- Deals with traffic planning and design of roads, of frontage development and of parking facilities and with the control of traffic to provide safe, convenient and economic movement of vehicles and pedestrians.
- Used to either improve an existing situation or, in the case of a new facility, to ensure that the facility is correctly and safely designed and adequate for the demands that will be placed on it.

Game theoretical approaches

- Infrastructure
- Users
- Cooperative environments

A game theory model of urban public traffic networks.

Su, B. B., Chang, H., Chen, Y. -Z., & He, D. R. (2007).

- Three players: public traffic company, the passengers and the government traffic management agency .

	Next Station	Next Line
Public traffic company	Max a/(lh)	a'*s'/T'
Passengers	Min a/(lh)	h'/s'
Government traffic management agency	Min alh	1/a'h's'

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$$a(i, t+1) = a(i, t) \times \left[1 - k(i, t) / \sum_{i=1}^{N} k(i, t)\right], \quad b(i, j, t+1) = b(i, j, t) \times \left[1 + l(i, j, t) / \sum_{i=1, j=1}^{N, N} l(i, j, t)\right]$$

- K denotes the degree of the bus station (how many stations a passenger can reach directly from this station without changing bus)
- l denotes the number of multiple edges between a pair of stations (the number of bus lines going through)
- i is an integer denoting a bus station,
- N denotes the total number of stations.
- The evolution of the node weight and the edge weight are due to the general fact that the traffic of the new bus line decrease number of waiting passengers and increase the congestion possibility along the line.

Micro-foundations of congestion and pricing: A game theory perspective

Levinson, D. (2005).

- Define penalties for early arrival (E), late arrival (L) and journey delay (D).

Payoff matrix

Vehicle 1	Vehicle 2		
	Early	On-time	Late
Early	$[0.5 * (E + D), 0.5 * (E + D)]$	$[E, 0]$	$[E, L]$
On-time	$[0, E]$	$[0.5 * (L + D), 0.5 * (L + D)]$	$[0, L]$
Late	$[L, E]$	$[L, 0]$	$[L + 0.5 * (L + D), L + 0.5 * (L + D)]$

Micro-foundations of congestion and pricing: A game theory perspective

Levinson, D. (2005).

Payoff matrix with congestion pricing

Vehicle 1	Vehicle 2		
	Early	On-time	Late
Early	$[0.5 * (E + D) + \tau_e, 0.5 * (E + D) + \tau_e]$	$[E, 0]$	$[E, L]$
On-time	$[0, E]$	$[0.5 * (L + D) + \tau_o, 0.5 * (L + D) + \tau_o]$	$[0, L]$
Late	$[L, E]$	$[L, 0]$	$[L + 0.5 * (L + D) + \tau_l, L + 0.5 * (L + D) + \tau_l]$
Early	$[0.5 * (E + D) + \mathrm{MAX}(0.5 * (D - E), 0), 0.5 * (E + D) + \mathrm{MAX}(0.5 * (D - E), 0)]$	$[E, 0]$	$[E, L]$
On-time	$[0, E]$	$[(L + D), (L + D)]$	$[0, L]$
Late	$[L, E]$	$[L, 0]$	$[2L + D, 2L + D]$

Micro-foundations of congestion and pricing: A game theory perspective
Levinson, D. (2005).

- Incremental social cost (ISC) = total cost
- The incremental private cost (IPC) = the additional amount each player pays in the absence of tolls
- Toll = ISC-IPC

Micro-foundations of congestion and pricing: A game theory perspective

Levinson, D. (2005).

Two-player game summary table

E, D, L	Number of Nash equilibria (unpriced)	Solutions (unpriced)	Total cost	Minimum total cost	Number of Nash equilibria (priced)	Solutions (priced)
0, 0, 0	9	all	0	0	9	All
0, 1, 0	6	*eo, el, oe, ol, le, lo*	0	0	6	*eo, el, oe, ol, le, lo*
0, 0, 1	3	*ee, eo, oe*	0	0	3	*ee, eo, oe*
0, 1, 1	2	*oe, eo*	0	0	2	*oe, eo*
1, 0, 0	4	*oo, ol, lo, ll*	0	0	4	*oo, ol, lo, ll*
1, 1, 0	2	*ol, lo*	0	0	2	*ol, lo*
1, 0, 1	1	*oo*	1	1	5	*eo, oe, oo, ol, lo*
1, 1, 1	5	*eo, oe, oo, ol, lo*	1 or 2	1	4	*eo, oe, ol, lo*
1, 2, 3	2	*eo, oe*	1	1	2	*eo, oe*
3, 1, 4	1	*oo*	5	3	2	*oe, eo*
4, 0, 3	1	*oo*	3	3	3	*oo, ol, lo*
4, 1, 3	1	*oo*	4	3	2	*ol, lo*
3, 0, 4	1	*oo*	4	3	2	*eo, oe*

Urban traffic control via Stackelber-Nash equilibria.

Alvarez, I., Alexander, V., & Poznyak, S. (2009).

- Focused on the traffic light control problem for urban traffic, using Game Theory and Extraproximal Method for its realization.
- A street can be seen as a finite capacity FIFO buffer or queue.
- Assume that the input flow is a Poisson Process with parameter $\lambda\xi$.
- Each player wants to minimize his penalties (in this case, the number of waiting cars) within the associated constraints.
- Since both aims are in conflict which can be resolved by the Nash-equilibrium concept .

Urban traffic control via Stackelber-Nash equilibria.

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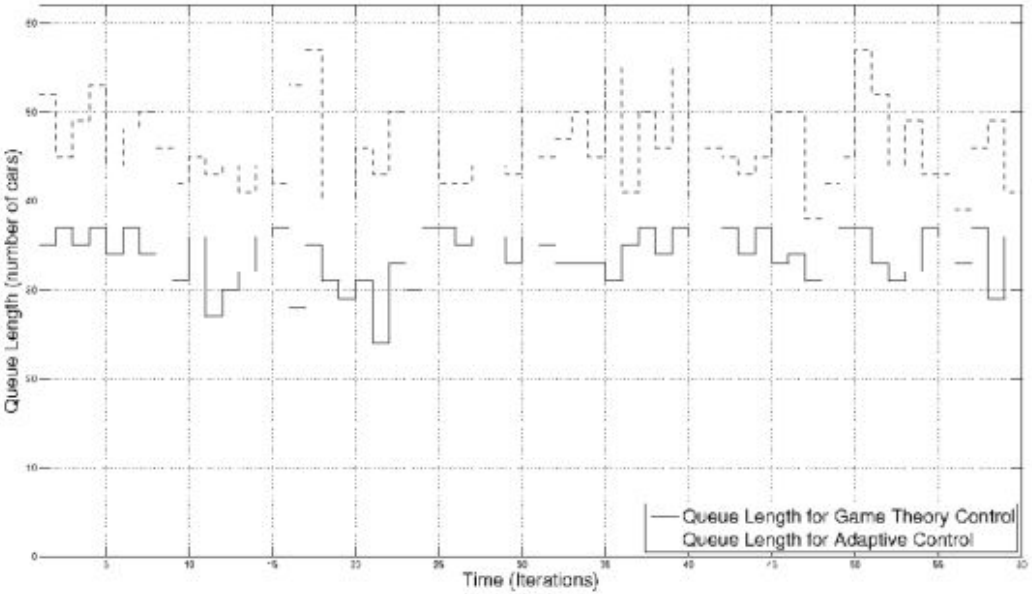


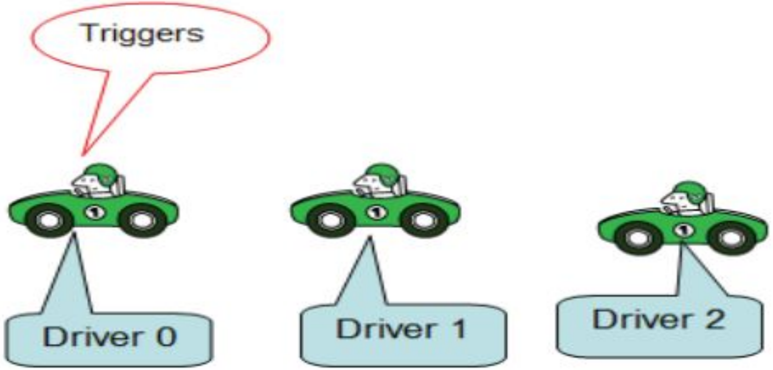
Evolutionary Game Theoretic Approach to Rear-End Events on Congested Freeway

Chatterjee, I., & Davis, G. A. (2013).

$$v_1(r_1 - h_1) \le -v_2(r_2 - h_2) + \left(\frac{v_0^2}{2a_0} - \frac{v_2^2}{2a_2}\right)$$

$$Y_c = \begin{cases} 0 & \text{if } r_1 + r_2 \le 2h + \dfrac{v}{2}\left[\dfrac{1}{a_0} - \dfrac{1}{a_{\text{hat}}}\right] \\ 1 & \text{otherwise} \end{cases}$$

$$V_{c2} = \sqrt{v^2 - 2a_{\text{hat}}\left[\frac{v^2}{2a_0} + v(2h - r_1 - r_2)\right]}$$



Evolutionary Game Theoretic Approach to Rear-End Events on Congested Freeway

Chatterjee, I., & Davis, G. A. (2013).

$$U_i = \begin{cases} U_0 - \lambda(r_i - r_0)^2 & \text{if } r_1 + r_2 \le 2h + \dfrac{v}{2}\left[\dfrac{1}{a_0} - \dfrac{1}{a_{\text{hat}}}\right] \\ U_0 - \lambda(r_i - r_0)^2 - \beta \times V_{c2} & \text{otherwise} \qquad \forall i = 1, 2 \end{cases}$$

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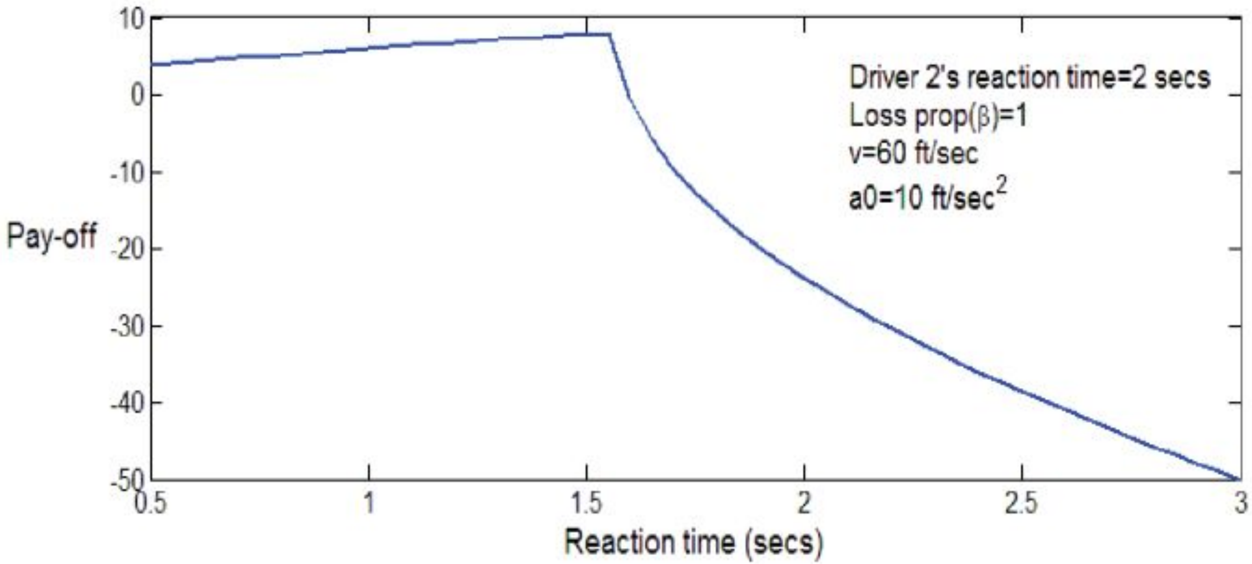

Evolutionary Game Theoretic Approach to Rear-End Events on Congested Freeway

Chatterjee, I., & Davis, G. A. (2013).

- An evolutionary stable strategy (ESS) is a strategy such that, if it is adopted by an entire population, no mutant (intruder) could successfully invade it.

$$u(y,x) \le u(x,x) \qquad \forall y \in \Delta$$

and

$$u(y,x) \le u(x,x) \Rightarrow u(y,y) < u(x,y)$$

Evolutionary Game Theoretic Approach to Rear-End Events on Congested Freeway

Chatterjee, I., & Davis, G. A. (2013).

	Driver 2	
Driver 1	A	I
A	6, 6	6, 10
I	10, 6	–590.5, –590.5

- Two pure Nash equilibria : (A, I) and (I, A)
- One mixed Nash Nash equilibrium : (A - 0.993, I - 0.007)
- (A, A) is not an ESS u(A,A) = 6 < u(I,A) = 10
- Crashes only occur between inattentive drivers and costs are allocated equally
- A population of purely attentive drivers is unstable. The main reason for such an observation is that a small fraction of inattentive drivers when confronted with attentive drivers can always get away with a higher payoff without being involved in a rear-end crash.

An intersection game-theory-based traffic control algorithm in a connected vehicle environment.

M. Elhenawy, A. A. Elbery, A. A. Hassan, & H. A. Rakha. (2015).

- Assumes that vehicles are equipped with Cooperative Adaptive Cruise Control (CACC) systems at uncontrolled intersections.
- CACC can obtain information through vehicle-to vehicle (V2V) and vehicle-to-infrastructure (V2I) communication and fuses it with the sensed information.
- Vehicle approaching the intersection reports its speed, location and direction. The intersection management center collect these information from all vehicles approaching the intersection and decide the action for each vehicles that will avoid crashes and give the lowest delay for each vehicle.

An intersection game-theory-based traffic control algorithm in a connected vehicle environment.

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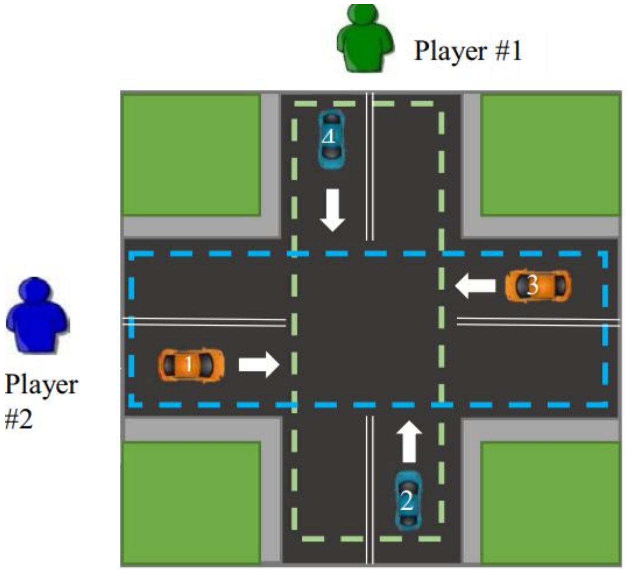
		Second driver	
		Swerve	Straight
First driver	Swerve	Tie, Tie	Lose, Win
	Straight	Win, Lose	Lose, Lose

(a)

		Second driver	
		Swerve	Straight
First driver	Swerve	0, 0	-1, 1
	Straight	1, -1	-100, -100

(b)



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1. Whenever a vehicle gets close to the central controller agent , it sends its current speed and position to the controller.
2. The controller chooses the nearest vehicle in each approach to the stop line, and based on their speeds it finds the set of feasible actions for each vehicle.
3. The controller gets each player's actions by cross multiplying its vehicle actions.
4. The controller sets up a game matrix for the current four vehicles.
5. The controller scans the matrix and for each action set of player #1 and player #2, it runs a simulation.
6. The controller solves the game matrix and reaches the Nash equilibrium
7. The controller sends back to each vehicle its optimum action.

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Player's Action	Utility if no conflict	Utility if conflict
(accelerate, accelerate)	4	-100
(constant, accelerate)	3	-100
(decelerate, accelerate)	2	-100
(accelerate, constant)	3	-100
(constant, constant)	2	-100
(decelerate, constant)	1	-100
(accelerate, decelerate)	2	-100
(constant, decelerate)	1	-100
(decelerate, decelerate)	0	-100

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Player #2

Player #1

	(accelerate, accelerate)	(constant, accelerate)	(accelerate, constant)	(constant, constant)
(accelerate, accelerate)	4,*4*	-100, *-100*	4,*3*	4,*2*
(constant, accelerate)	-100, *-100*	-100, *-100*	-100, *-100*	3,*2*
(accelerate, constant)	-100, *-100*	-100, *-100*	-100, *-100*	3,*2*
(constant, constant)	-100, *-100*	-100, *-100*	-100, *-100*	-100, *-100*

Future

- Need more research on how to utilize these models in road traffic handling.
- World is moving towards autonomous vehicles. Therefore these models will be obsolete in such environment.
- In such scenario, will it be possible to use networking algorithms to in road traffic handling instead of these models?

Have you ever noticed that anybody driving slower than you is an idiot, and anyone going faster than you is a maniac?

-George Carlin

Thank You!

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