Single-Shot Interactions

CMPUT 654: Modelling Human Strategic Behaviour

Camerer, Ho, and Chong (2004)
McKelvey & Palfrey (1995)
Wright & Leyton-Brown (2017) [optional]

Lecture Outline

1. Camerer, Ho, and Chong (2004)
2. McKelvey & Palfrey (1995)
3. Wright & Leyton-Brown (2017)

Camerer, Ho, and Chong (2004)

Why:

- One of the most influential papers on single-shot play
- Proposes a very intuitive model that also predicts well
- Shows some drawbacks of standard practice in behavioural economics

- Proposes the **cognitive hierarchy** model of human behaviour
- Presents experimental data in support

Fun Game:
Keynesian Beauty Contest

- Let's play the Beauty Contest game!
- Everyone chooses an **integer between 0 and 100**
- Whoever is closest to **2/3 of the average** wins

Iterative Strategic Thinking

Level-0: Some **nonstrategic** distribution of play
(uniform randomization, truthfulness, maxmin, etc.)

Level-1: Respond to **level-0** players

Level-2: Respond to **level-1**, or to levels 0,1

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Level-*k*: respond to **level *k*-1**, or to levels 0,1,...,*k*-1

Cognitive Hierarchy

- Levels **distributed** according to $g(k)$
- Level-k responds to distribution $g(m \mid m < k)$
  - Every agent **wrongly** believes that all the other agents perform **fewer steps of reasoning**
  - But every agent gets the **conditional distribution** right
- Distribution is a **parameter** of the model to be fit from data
  - This paper uses single-parameter Poisson(τ)

Model Fit

TABLE IV

MODEL FIT (LOG-LIKELIHOOD LL AND MEAN SQUARED DEVIATION MSD)

Data set	Stahl and Wilson	Cooper and Van Huyck	Costa-Gomes et al.	Mixed	Entry
Log-likelihood					
Cognitive hierarchy (Game-specific τ)	−360	−838	−264	−824	−150
Cognitive hierarchy (Common τ)	−458	−868	−274	−872	−150
Nash equilibrium[a]	−1823	−5422	−1819	−1270	−154
Mean squared deviation					
Cognitive hierarchy (Game-specific τ)	0.0074	0.0090	0.0035	0.0097	0.0004
Cognitive hierarchy (Common τ)	0.0327	0.0145	0.0097	0.0179	0.0005
Nash equilibrium	0.0882	0.2038	0.1367	0.0387	0.0049

a. The Nash Equilibrium result is derived by allowing a nonzero mass of 0.0001 on nonequilibrium strategies.

Parameter Estimates

TABLE III

PARAMETER ESTIMATE τ FOR COGNITIVE HIERARCHY MODELS

Data set	Stahl and Wilson	Cooper and Van Huyck	Costa-Gomes et al.	Mixed	Entry
Game-specific τ					
Game 1	2.93	15.90	2.28	0.98	0.70
Game 2	0.00	1.07	2.27	1.71	0.85
Game 3	1.40	0.18	2.29	0.86	—
Game 4	2.34	1.28	1.26	3.85	0.73
Game 5	2.01	0.52	1.80	1.08	0.70
Game 6	0.00	0.82	1.67	1.13	
Game 7	5.37	0.96	0.88	3.29	
Game 8	0.00	1.54	2.18	1.84	
Game 9	1.35		1.89	1.06	
Game 10	11.33		2.26	2.26	
Game 11	6.48		1.23	0.87	
Game 12	1.71		1.03	2.06	
Game 13			2.28	1.88	
Game 14				9.07	
Game 15				3.49	
Game 16				2.07	
Game 17				1.14	
Game 18				1.14	
Game 19				1.55	
Game 20				1.95	
Game 21				1.68	
Game 22				3.06	
Median τ	1.86	1.01	1.89	1.77	0.71
Common τ	1.54	0.82	1.73	1.48	0.73

Economic Value

TABLE VIII
ECONOMIC VALUE OF VARIOUS THEORIES

Data set	Stahl and Wilson	Cooper and Van Huyck	Costa-Gomes et al.	Mixed	Entry
Observed payoff	195	586	264	328	118
Clairvoyance payoff	243	664	306	708	176
Economic value					
Clairvoyance	48	78	42	380	58
Cognitive hierarchy (Common τ)	13	55	22	132	10
Nash equilibrium	5	30	15	−17	2
% Maximum economic value achieved					
Cognitive hierarchy (Common τ)	26%	71%	52%	35%	17%
Nash equilibrium	10%	39%	35%	−4%	3%

The economic value is the total value (in experimental payoffs) of all rounds that a "hypothetical" subject will earn using the respective model to predict other's behavior and best responds with the strategy that yields the highest expected payoff in each round.

Fun Game: Stag Hunt

	Stag	Hare
Stag	2,2	0,1
Hare	1,0	1,1

- Two hunters must independently decide whether to hunt for stag or hare
- **Stags** are more valuable, but more difficult to catch; the hunt will only succeed if **both hunters** participate
- **Hares** can be caught by a single hunter acting alone, but are **less valuable**

Anomalies Explained

1. Behaviour in market-entry games
   entry monotonic in demand

2. Limited steps of iterated removal of dominated strategies
   James: but c.f. the Traveller's Dilemma

3. Risk-dominant vs. Payoff-dominant equilibria:
   more people play risk-dominant strategies as number of players increase

Economic Implications

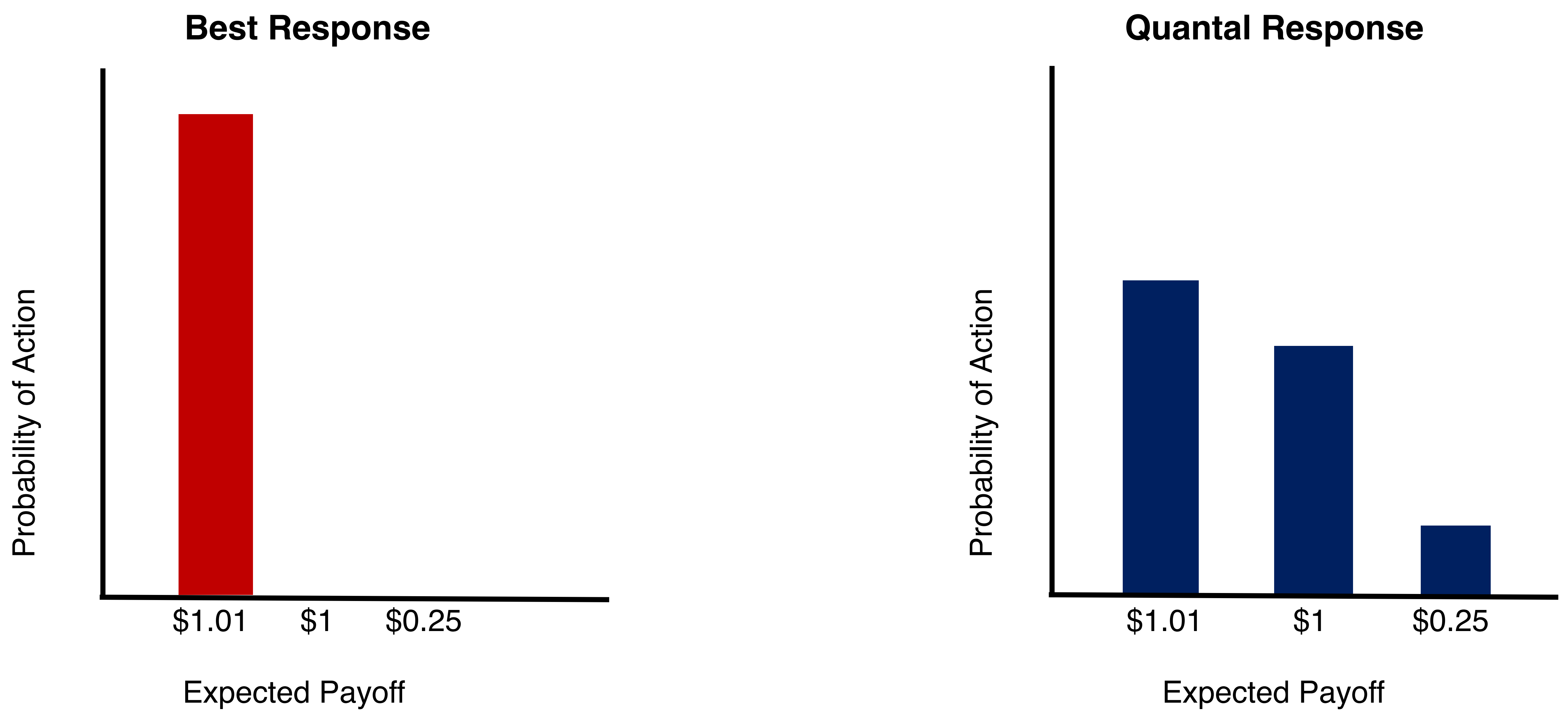
1. Speculation
2. Money Illusion

McKelvey & Palfrey (1995)

Why:
Origin of "the other" family of behavioural models for single-shot games.

- Introduces **quantal response equilibrium**, which generalizes Nash equilibrium
    - All agents "quantally respond" to each other simultaneously rather than best responding
- Also experimental data

Quantal Response



- **Best response**: Maximum utility action is always played
- **Quantal response**: High-utility actions played often, low-utility actions played rarely

Quantal Response Equilibrium

- In a Nash equilibrium, every player **best responds** to all others
- In a quantal response equilibrium, every player **quantally responds** to all others
- No single **functional form** for quantal response
  - Anything that yields higher probability for higher EU
  - In practice, usually **softmax**: $s_i(a_i) = \text{softmax}\left(u_i(a_i, s_{-i})\right)$

$$= \frac{\exp\left[\lambda u_i(a_i, s_{-i})\right]}{\sum_{a_i' \in A_i} \exp\left[\lambda u_i(a_i', s_{-i})\right]}$$

Mathematical Properties of QRE

1. **Theorem 1:** QRE always exists

2. **Theorem 3:** Unique branch of $\pi^*(\lambda)$ starting from λ=0 and converging to a **unique Nash equilibrium** as $\lambda \to \infty$

3. **Example:** Not every limit logit equilibrium is trembling-hand perfect

Data (aggregated)

TABLE IV
DATA AND ESTIMATES FOR O'NEILL

	Number	Frequency	Rand	NE	QRE
A_1	949	0.362	0.250	0.400	0.360
A_2	579	0.221	0.250	0.200	0.213
A_3	565	0.215	0.250	0.200	0.213
A_4	532	0.203	0.250	0.200	0.213
B_1	1119	0.426	0.250	0.400	0.426
B_2	592	0.226	0.250	0.200	0.191
B_2	470	0.179	0.250	0.200	0.191
B_4	444	0.169	0.250	0.200	0.191
λ			0	∞	1.313
$-\mathfrak{L}^*$			7278	7016	7004

Data (by period)

TABLE V

DATA AND ESTIMATES FOR O'NEILL EXPERIMENTS, BROKEN DOWN BY PERIOD

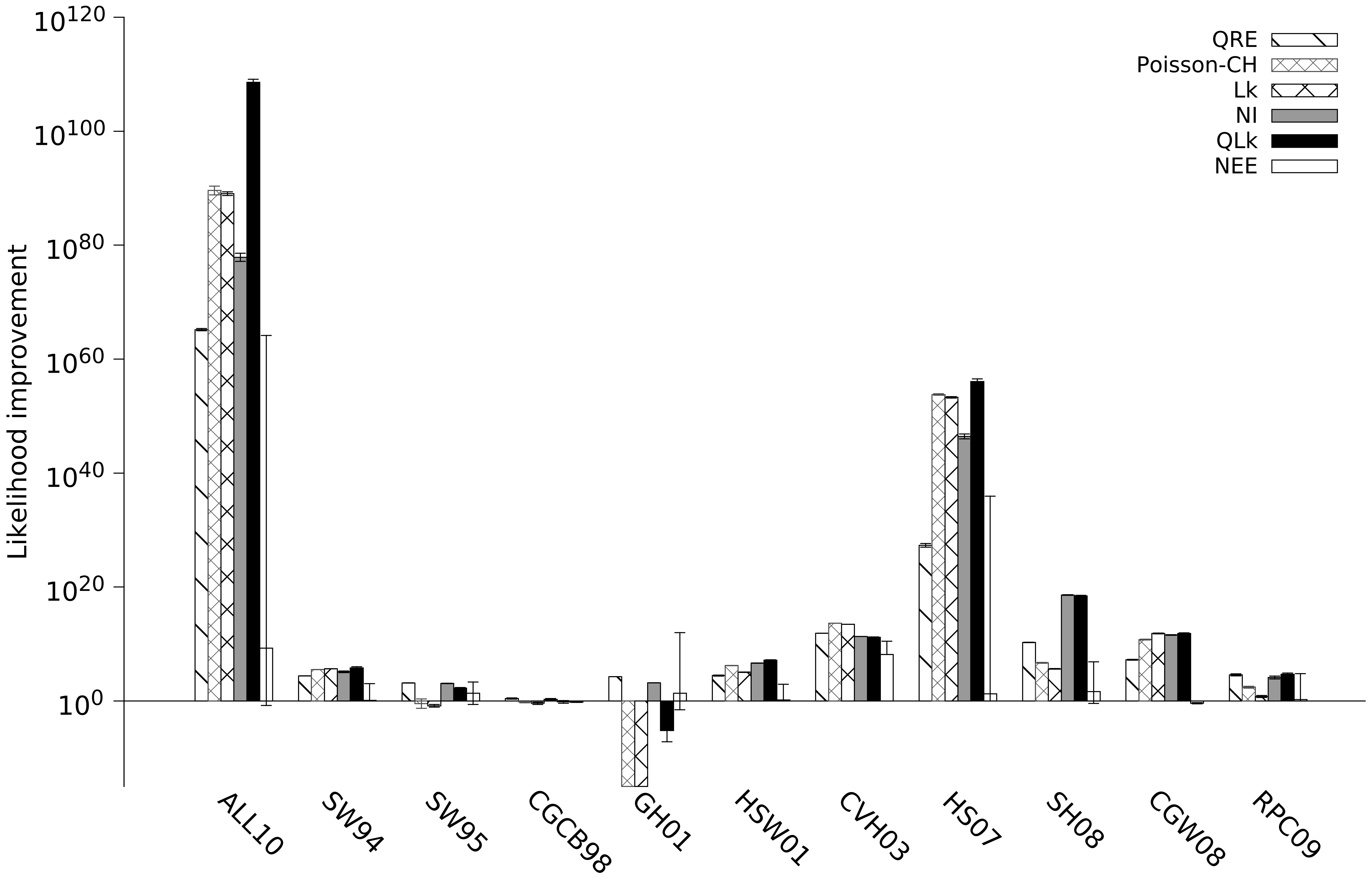
											$-\mathfrak{L}^*$		
Periods		A_1	A_2	A_3	A_4	B_1	B_2	B_3	B_4	λ	QRE	Nash	Rand
1–15	Actual	0.363	0.208	0.227	0.203	0.445	0.211	0.179	0.165	1.262	995	997	1040
	Predicted	0.358	0.214	0.214	0.214	0.427	0.191	0.191	0.191				
16–30	Actual	0.349	0.187	0.229	0.234	0.421	0.221	0.181	0.176	1.120	1004	1007	1040
	Predicted	0.352	0.216	0.216	0.216	0.429	0.190	0.190	0.190				
31–45	Actual	0.376	0.205	0.216	0.203	0.400	0.213	0.200	0.187	3.313	1005	1005	1040
	Predicted	0.385	0.205	0.205	0.205	0.413	0.196	0.196	0.196				
46–60	Actual	0.331	0.237	0.216	0.216	0.424	0.216	0.187	0.173	0.798	1006	1011	1040
	Predicted	0.332	0.223	0.223	0.223	0.433	0.189	0.189	0.189				
61–75	Actual	0.347	0.227	0.211	0.216	0.432	0.227	0.165	0.176	1.034	1002	1005	1040
	Predicted	0.348	0.217	0.217	0.217	0.430	0.190	0.190	0.190				
76–90	Actual	0.379	0.248	0.208	0.165	0.435	0.219	0.163	0.184	1.823	994	996	1040
	Predicted	0.372	0.209	0.209	0.209	0.420	0.193	0.193	0.193				
91–105	Actual	0.387	0.232	0.200	0.181	0.427	0.272	0.179	0.123	2.482	995	996	1040
	Predicted	0.380	0.207	0.207	0.207	0.416	0.195	0.195	0.195				

Note. The first 15 periods were practice rounds.

Wright & Leyton-Brown (2017)

- Large-scale comparison of different behavioural models
- Combines data-sets from 10 different studies

Prediction Performance



Model Variations