# Convolutional Neural Networks

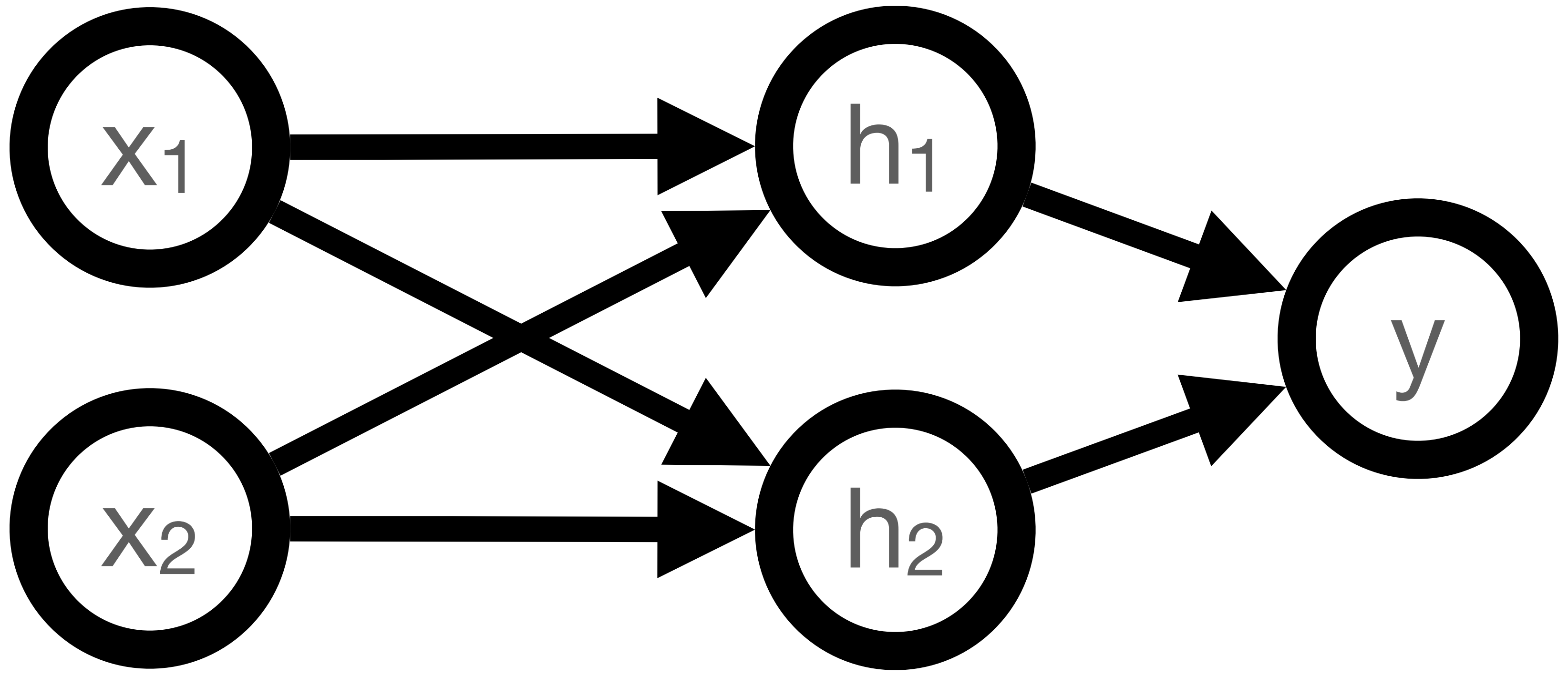
CMPUT 366: Intelligent Systems

GBC §9.0-9.4

# Logistics

- Midterm is next **Monday, March 15**
  - Exam delivered via eclass
  - 24 hour window to take the exam
- There will be a review class this Friday
  - More details about format, likely questions, etc.

# Recap: Feedforward Neural Network




$$h_1(\mathbf{x}; \mathbf{w}^{(1)}, b^{(1)}) = g\left(b^{(1)} + \sum_{i=1}^{n} w_i^{(1)} x_i\right)$$

$$y(\mathbf{x}; \mathbf{w}, \mathbf{b}) = g\left(b^{(y)} + \sum_{i=1}^{n} w_i^{(y)} h_i(\mathbf{x}_i; \mathbf{w}^{(i)}, b^{(i)})\right)$$

$$= g\left(b^{(y)} + \sum_{i=1}^{n} w_i^{(y)} g\left(b^{(i)} + \sum_{j=1}^{n} w_j^{(i)} x_j\right)\right)$$

- A **neural network** is many **units** **composed** together
- **Feedforward neural network:** Units arranged into **layers**
    - Each layer takes outputs of **previous layer** as its **inputs**

# Recap: Training Neural Networks

- Specify a **loss** $L$ and a set of **training examples:**

$$E = (\mathbf{x}^{(1)}, y^{(1)}), \ldots, (\mathbf{x}^{(n)}, y^{(n)})$$

- Training by **gradient descent**:

Loss function (e.g., squared error)

1. Compute **loss** on training data: $L(\mathbf{W}, \mathbf{b}) = \sum_i \ell\left(f(\mathbf{x}^{(i)}; \mathbf{W}, \mathbf{b}), y^{(i)}\right)$

   Prediction — Target

2. Compute **gradient** of loss: $\nabla L(\mathbf{W}, \mathbf{b})$

3. **Update parameters** to make loss smaller:

$$\begin{bmatrix} \mathbf{W}^{new} \\ \mathbf{b}^{new} \end{bmatrix} = \begin{bmatrix} \mathbf{W}^{old} \\ \mathbf{b}^{old} \end{bmatrix} - \eta \nabla L(\mathbf{W}^{old}, \mathbf{b}^{old})$$

# Recap: Automatic Differentiation

- **Forward mode** sweeps through the graph, computing $s_i' = \frac{\partial s_i}{\partial s_1}$ for each $s_i$
  - The **numerator varies**, and the **denominator is fixed**
  - At the end, we have computed $s_n' = \frac{\partial s_n}{\partial x_i}$ for a **single** input $x_i$
- **Backward mode** does the opposite:
  - For each $s_i$, computes the **local gradient** $\overline{s_i} = \frac{\partial s_n}{\partial s_i}$
  - The **numerator is fixed**, and the **denominator varies**
  - At the end, we have computed $\overline{x_i} = \frac{\partial s_n}{\partial x_i}$ for each input $x_i$
- **Key point:** The intermediate results are computed **numerically** at **each step**

# Lecture Outline

1. Recap
2. Neural Networks for Image Recognition
3. Convolutional Neural Networks

# Image Classification

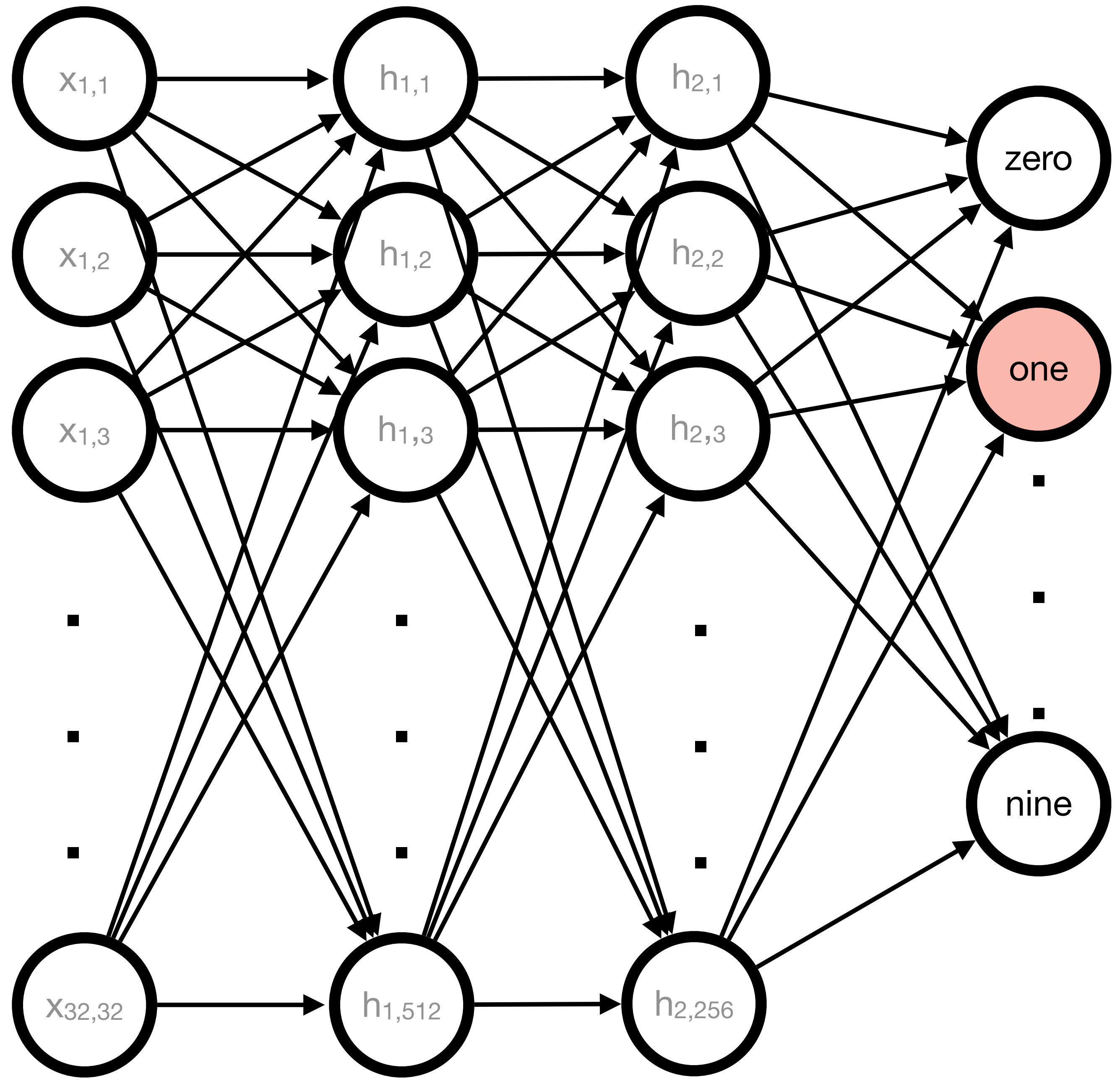

**Problem:** Recognize the handwritten digit from an image

- What are the **inputs**?
- What are the **outputs**?
- What is the **loss**?

# Image Classification with Neural Networks

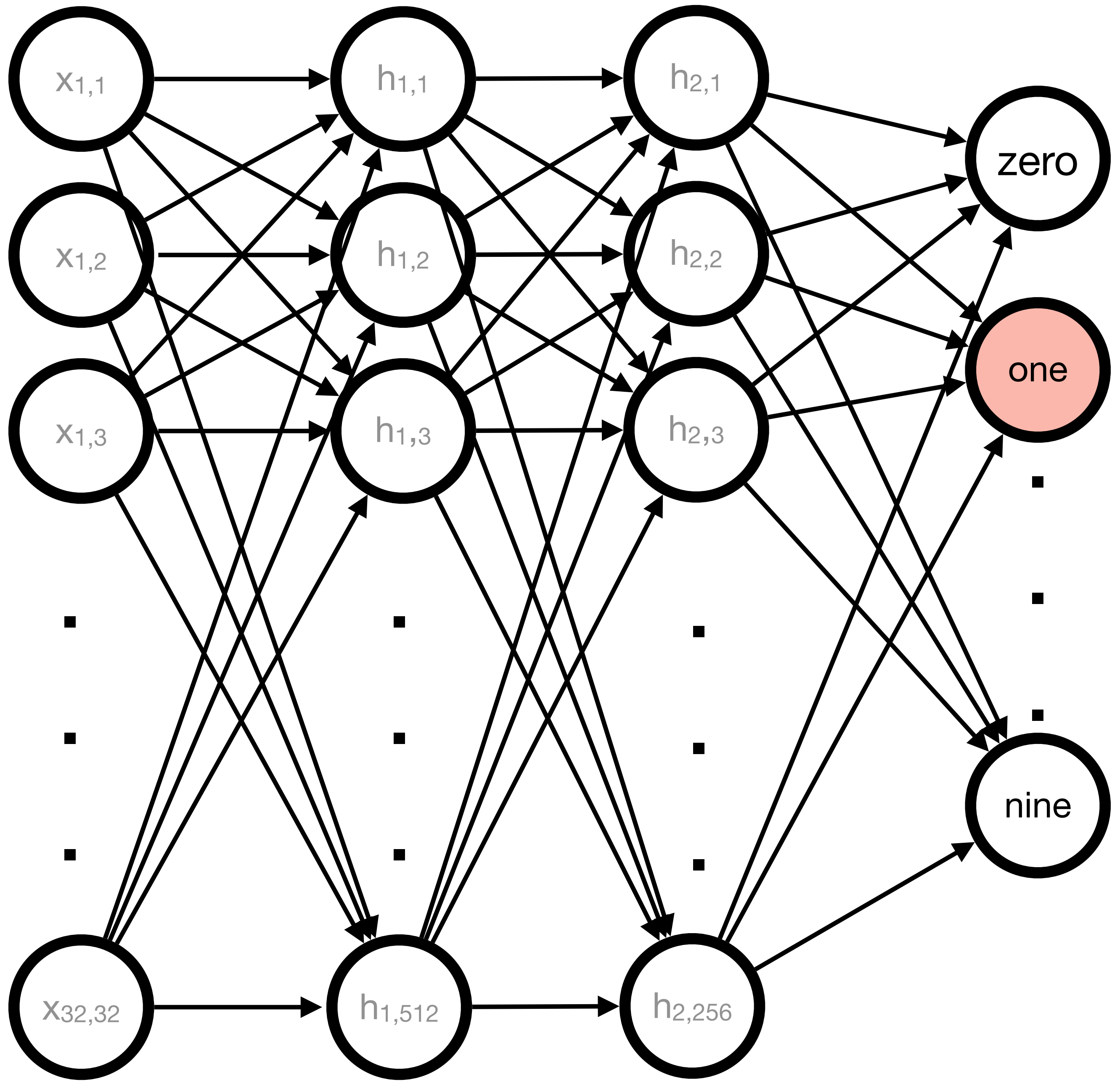
How can we use a **neural network** to solve this problem?

- How to represent the **inputs**?
- How to represent the **outputs**?
- What are the **parameters**?
- What is the **loss**?



# Image Recognition Issues

- For a large image, the number of parameters will be **very large**
  - For 32x32 greyscale image, hidden layer of 512 units hidden layer of 256 units, 1024×512 + 512×256 + 256×10 = **657,920 weights** (and 1802 offsets)
  - Needs **lots of data** to train
- Want to **generalize** over **transformations** of the input

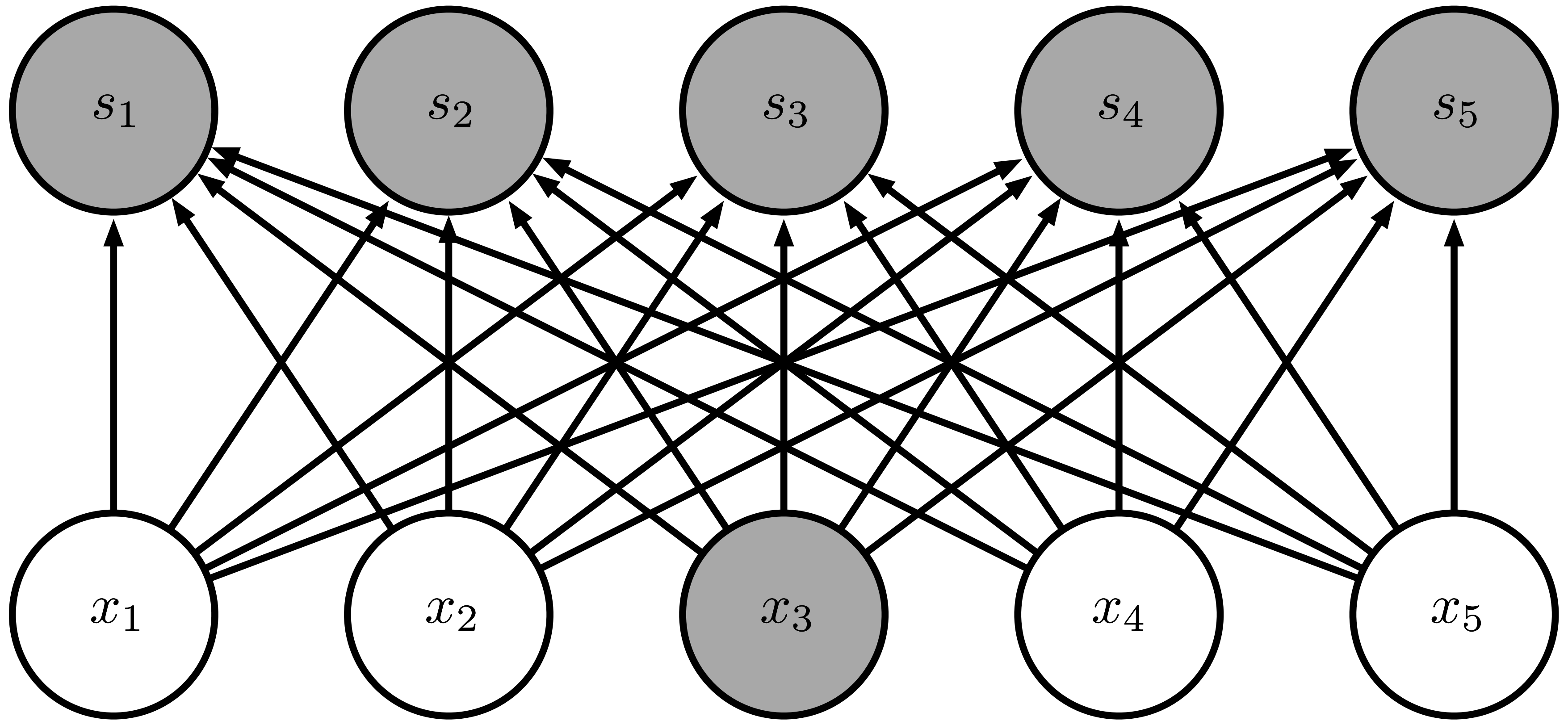



# Convolutional Neural Networks

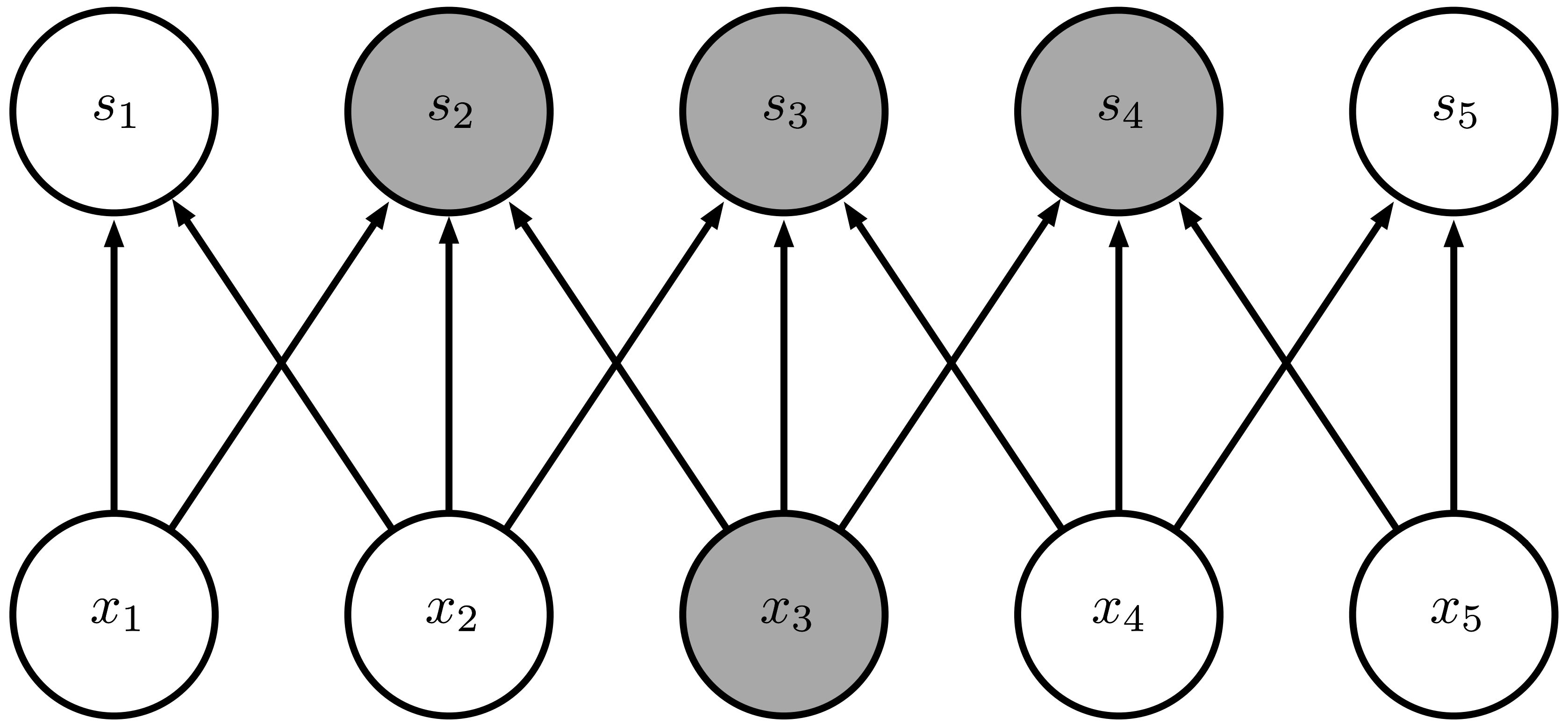
- **Convolutional neural networks:** a **specialized architecture** for image recognition
- Introduce two **new operations**:
    1. Convolutions
    2. Pooling
- Efficient **learning** via:
    1. Sparse interactions
    2. Parameter sharing
    3. Equivariant representations

# Sparse Interactions

Dense
connections



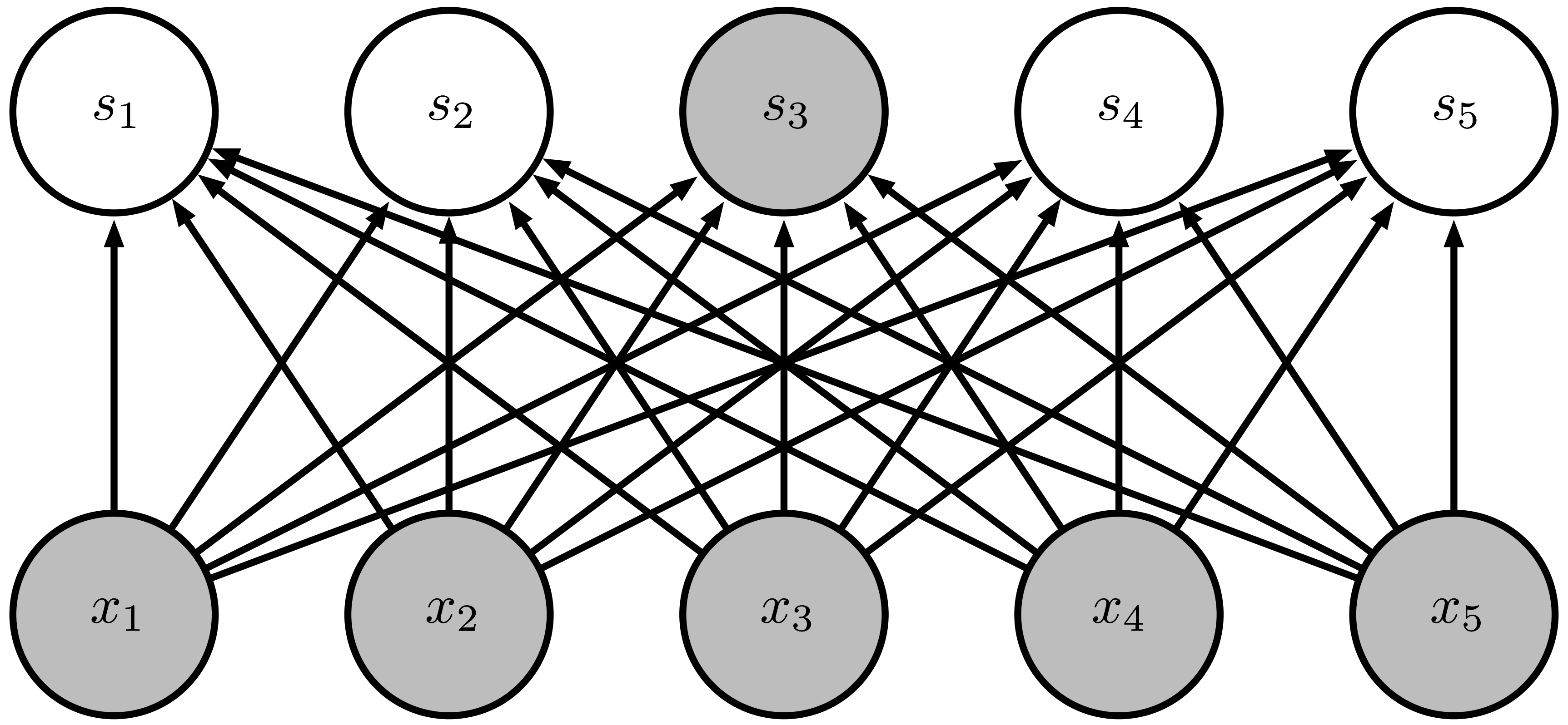

Sparse
connections



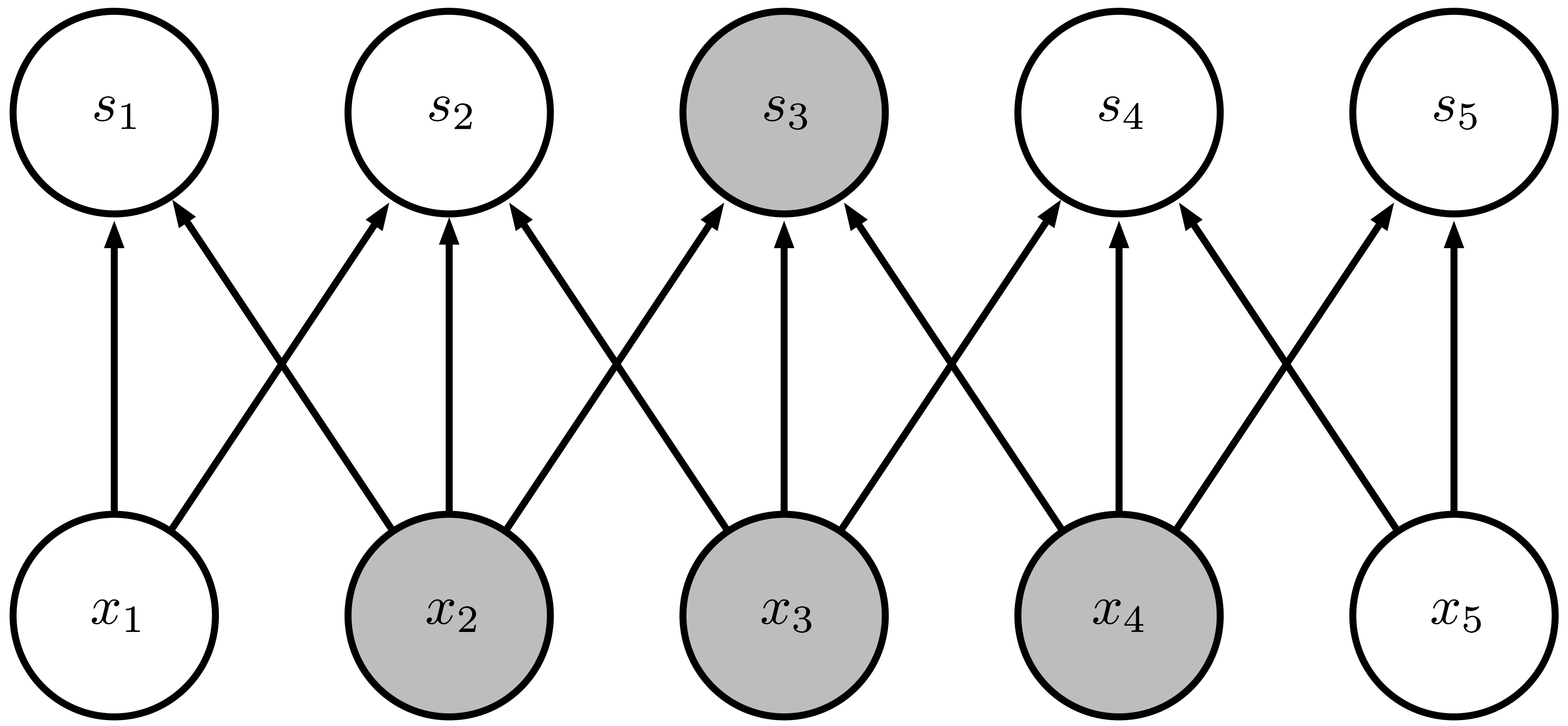

(Images: Goodfellow 2016)

# Sparse Interactions

Dense connections



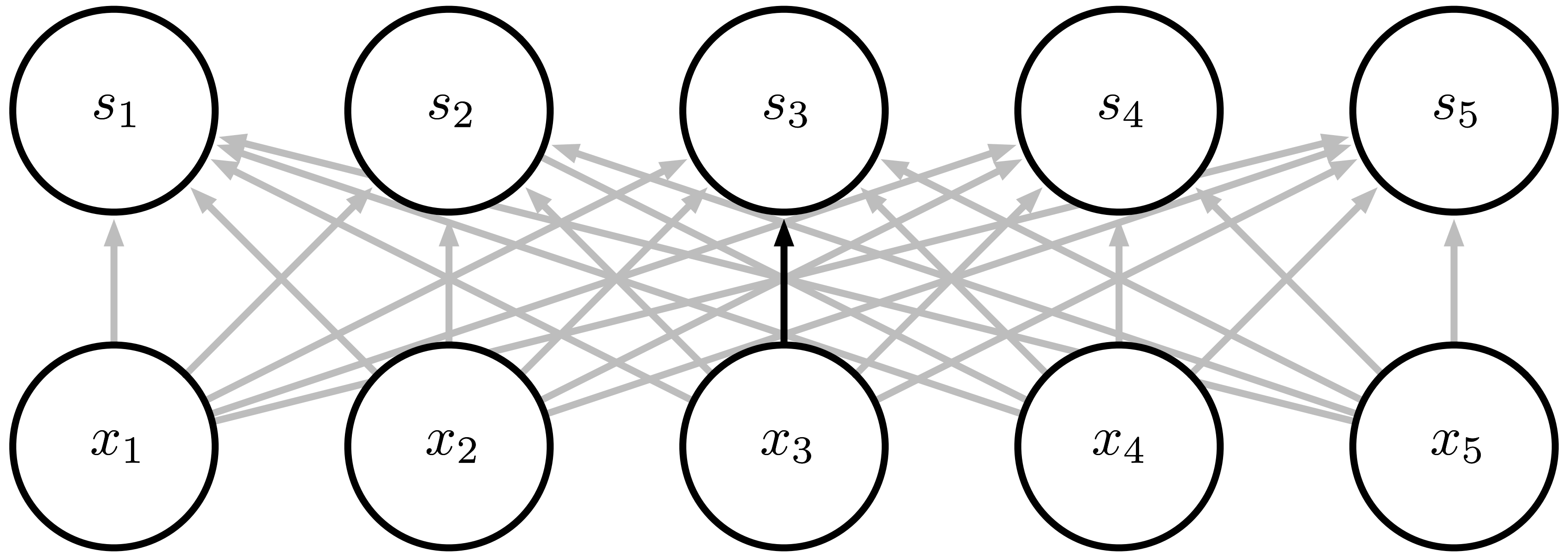

Sparse connections



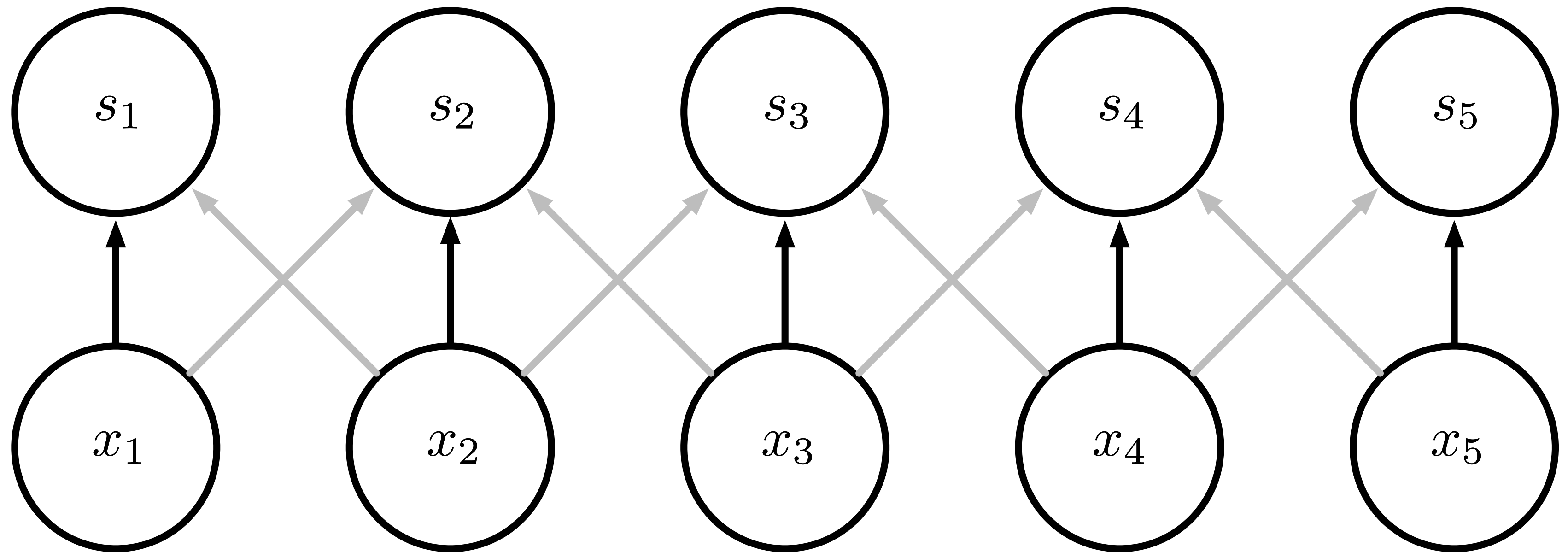

(Images: Goodfellow 2016)

# Parameter Sharing

Traditional neural nets learn a **unique value** for **each connection**



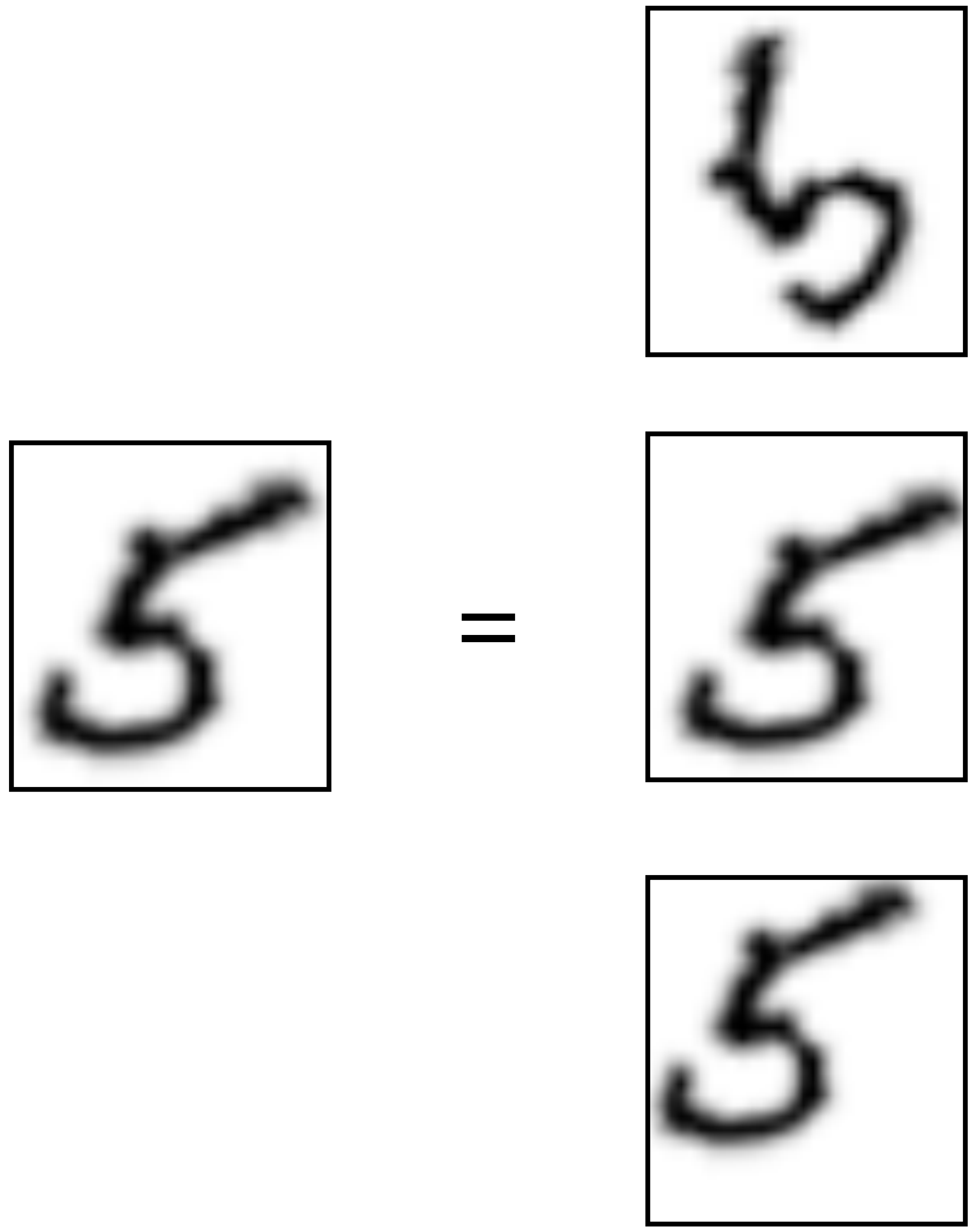

Convolutional neural nets **constrain** multiple parameters to be **equal**




(Images: Goodfellow 2016)

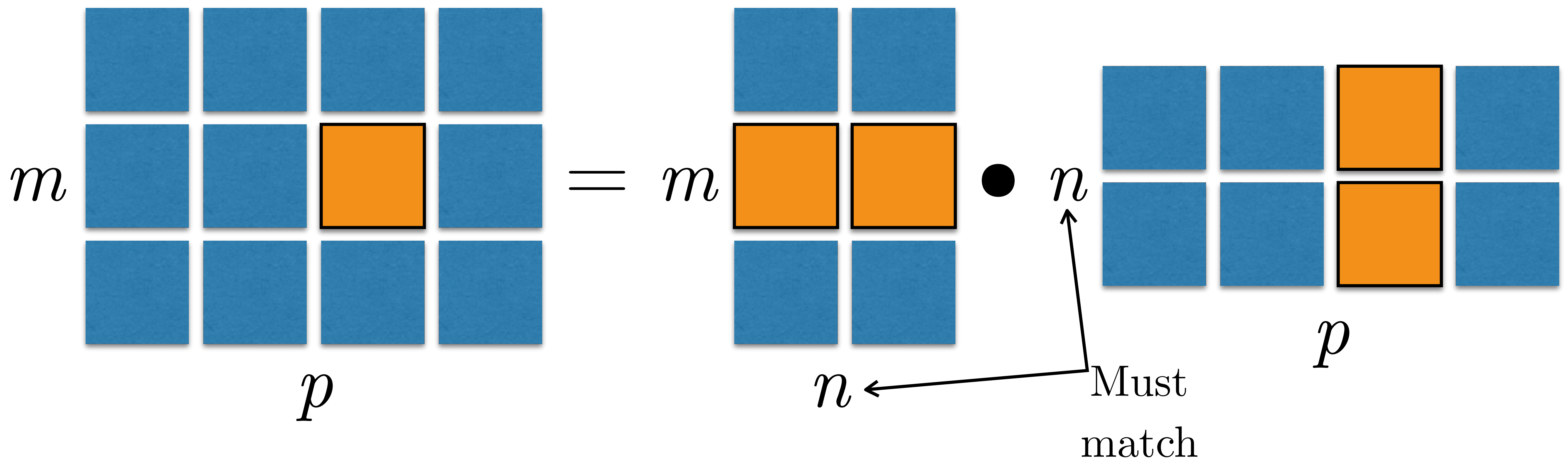
# Equivariant Representations

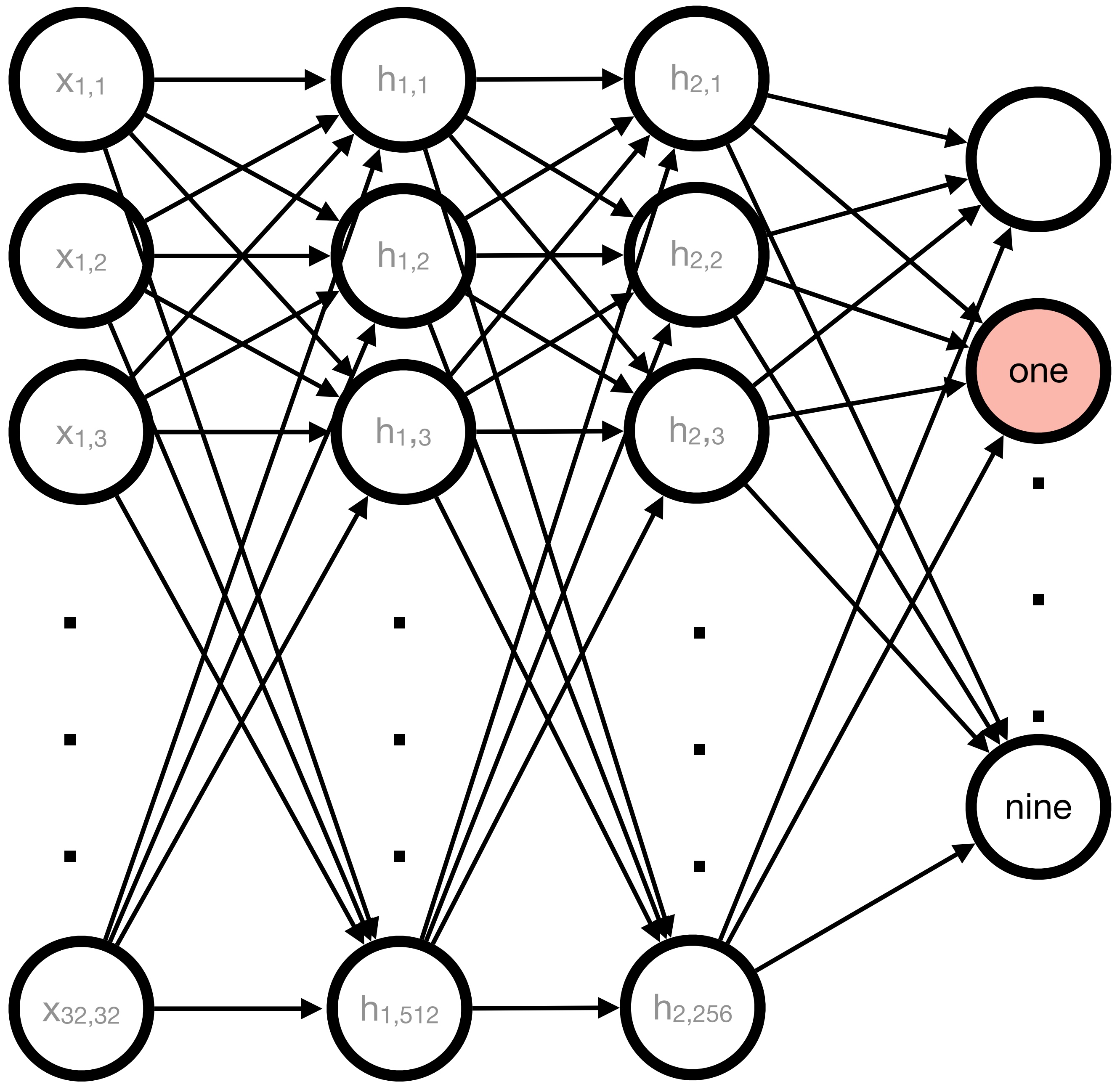
- We want to be able to recognize transformed versions of inputs we have seen before:
  - Translation (moved)
  - Rotation
- Without having been **trained** on all transformed versions



# Operation: Matrix Product

Recall that we can represent the **activations** in a neural network by a **matrix product**




$$\mathbf{h_1} = g_h\left(W^{(1)}\mathbf{x} + \mathbf{b}^{(1)}\right)$$

$$\mathbf{h_2} = g_h\left(W^{(2)}\mathbf{h_1} + \mathbf{b}^{(2)}\right)$$

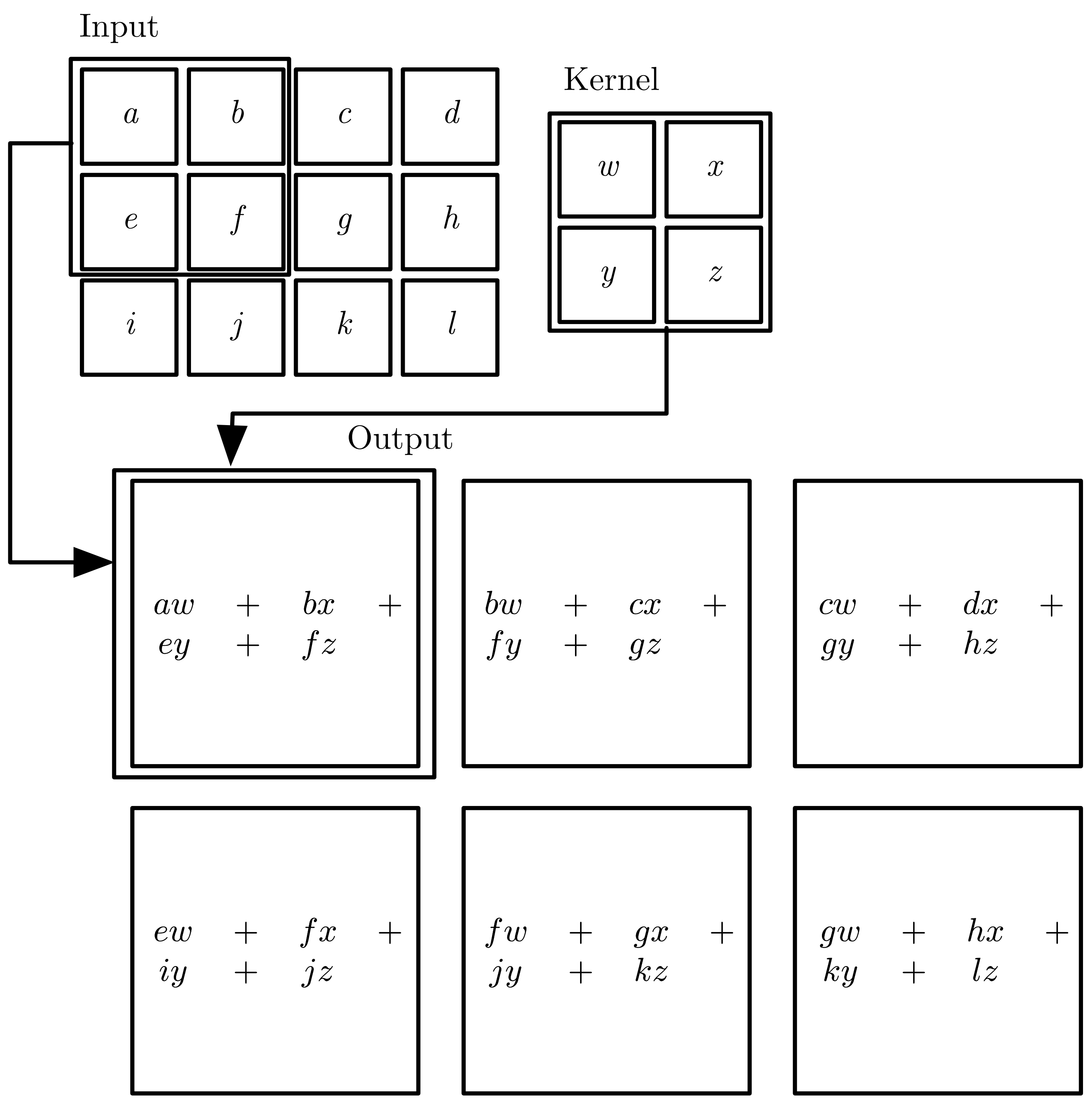
$$\mathbf{y} = g_y\left(W^{(3)}\mathbf{h_2} + \mathbf{b}^{(3)}\right)$$

(Image: Goodfellow 2016)

# Operation: 2D Convolution

Convolution scans a small block of weights (called the **kernel**) over the elements of the inputs, taking **weighted averages**

- Note that input and output dimensions **need not match**
- **Same weights** used for very many combinations



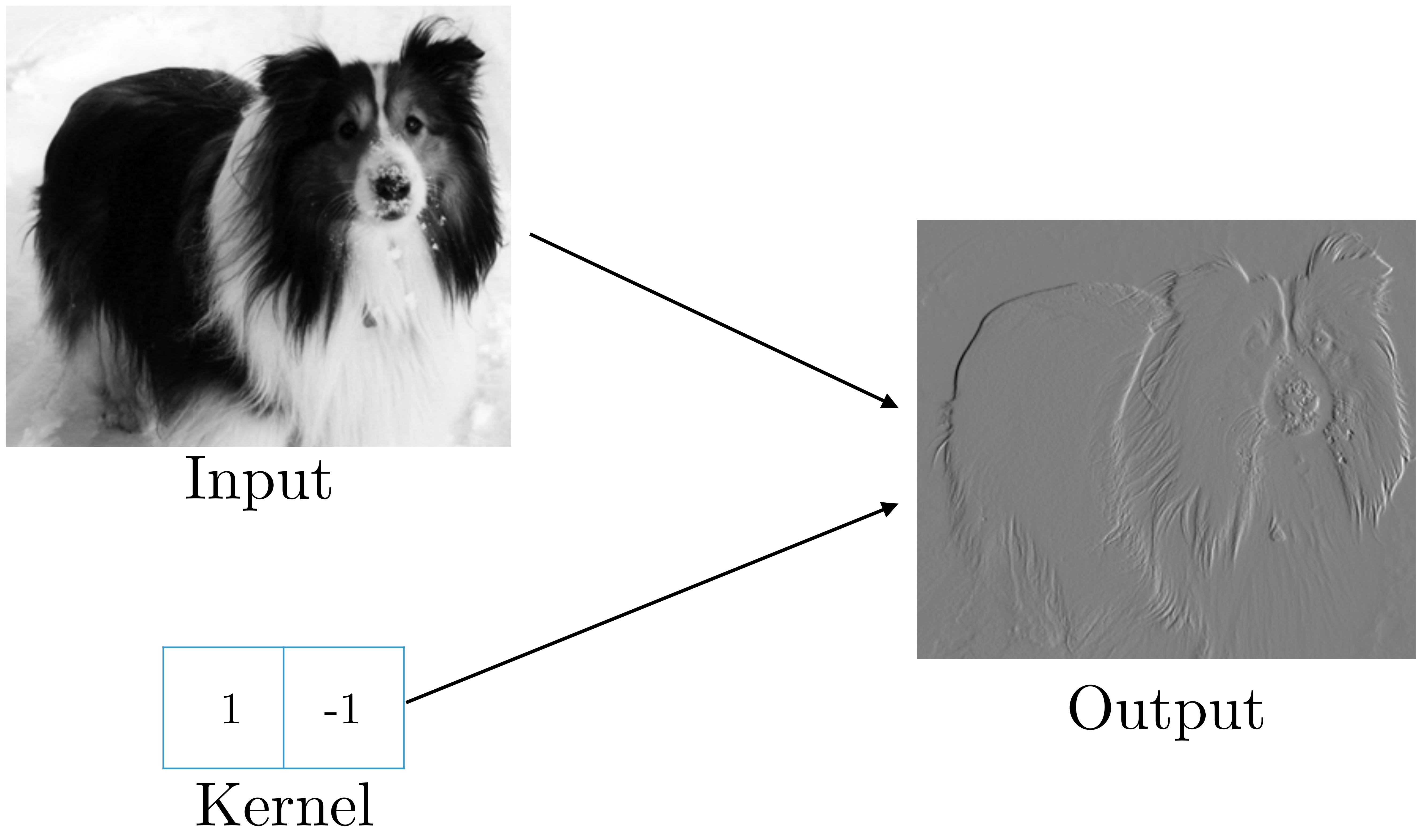

(Image: Goodfellow 2016)

# Replace Matrix Multiplication by Convolution

**Main idea:** Replace matrix multiplications with convolutions

- **Sparsity:** Inputs only combined with **neighbours**
- **Parameter sharing:** Same kernel used for entire input

# Example: Edge Detection




(Image: Goodfellow 2016)

# Efficiency of Convolution

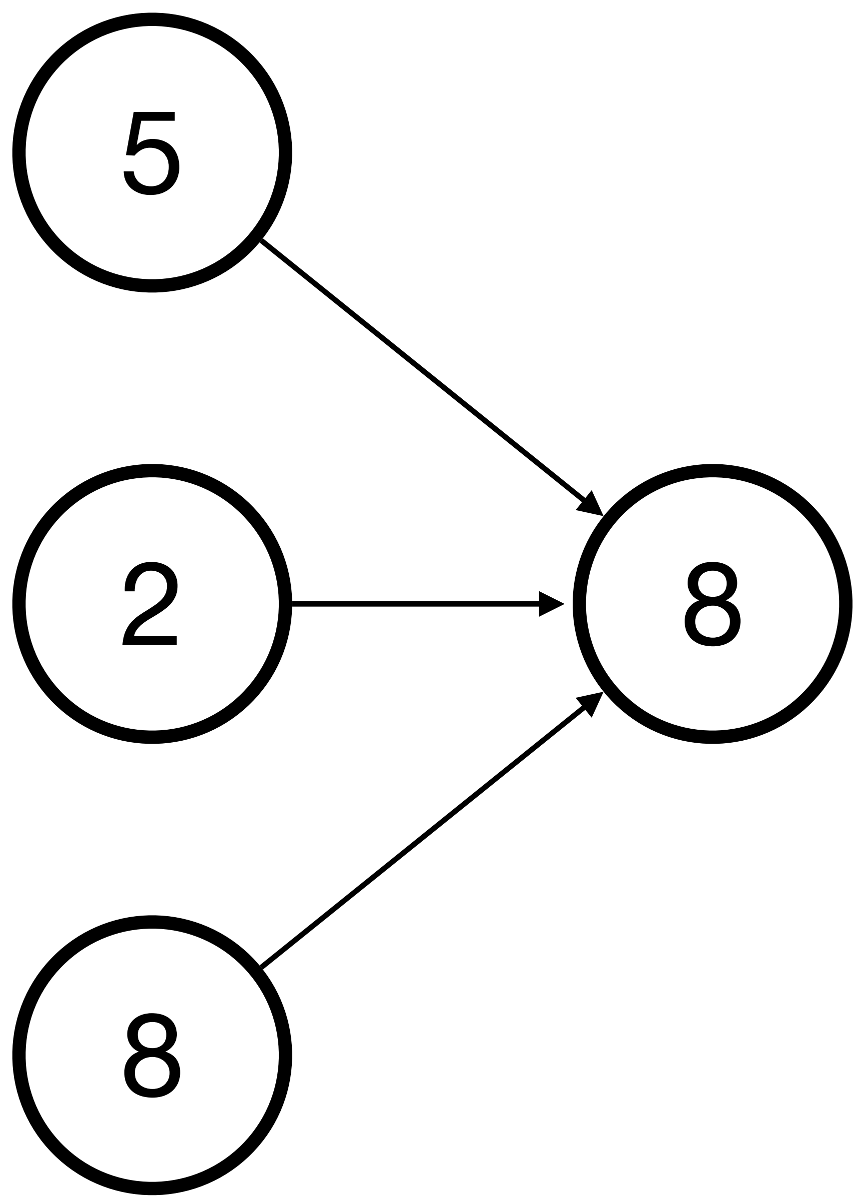
Input size: 320 by 280
Kernel size: 2 by 1
Output size: 319 by 280

| | Dense matrix | Sparse matrix | Convolution |
|---|---|---|---|
| **Stored floats** | 319*280*320*280 > 8e9 | 2*319*280 = 178,640 | 2 |
| **Float muls or adds** | > 16e9 | Same as convolution (267,960) | 319*280*3 = 267,960 |

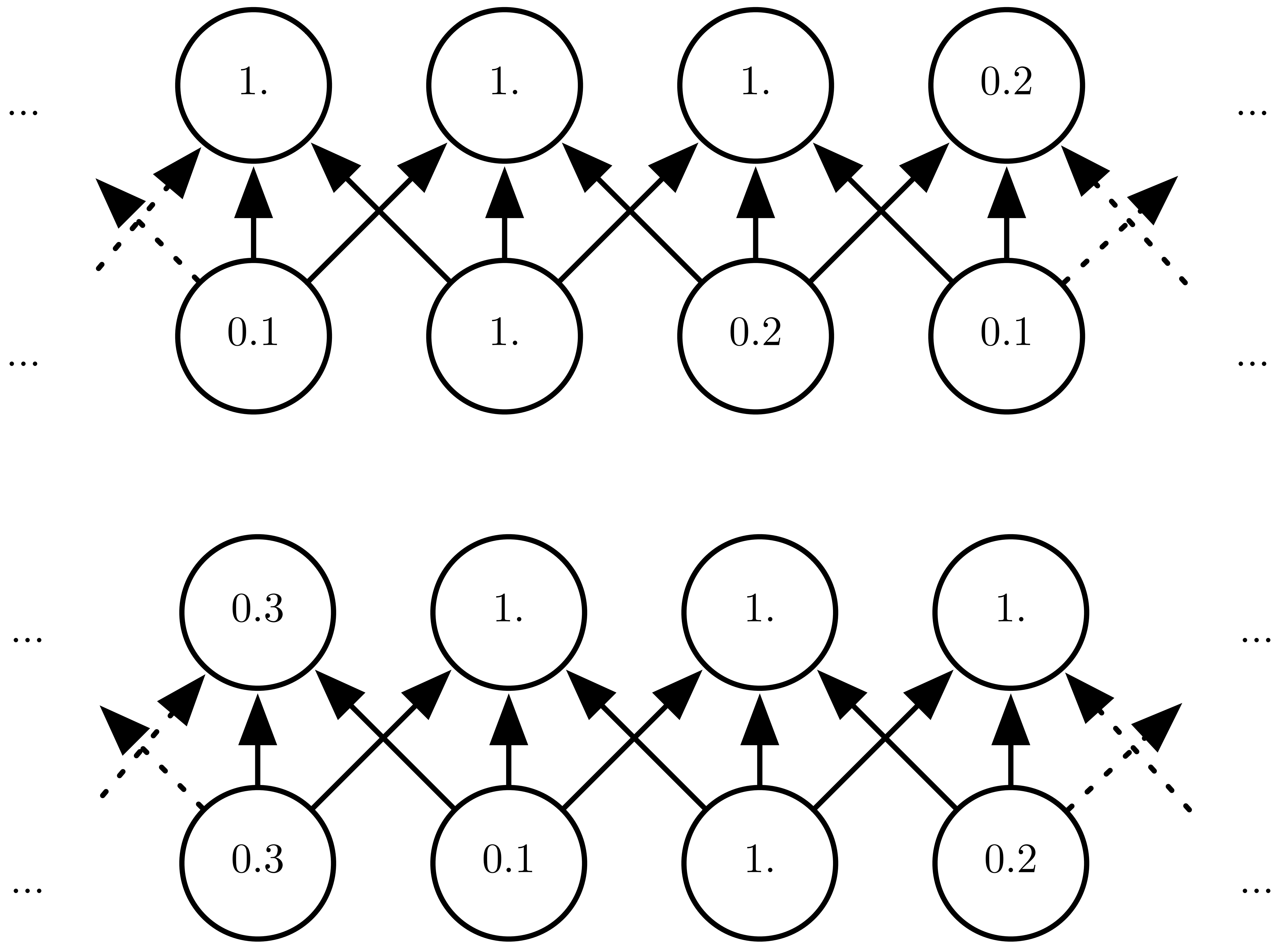
(Goodfellow 2016)

# Operation: Pooling

- Pooling **summarizes** its inputs into a single value, e.g.,
  - max
  - average
- Max-pooling is **parameter-free** (no bias or edge weights)

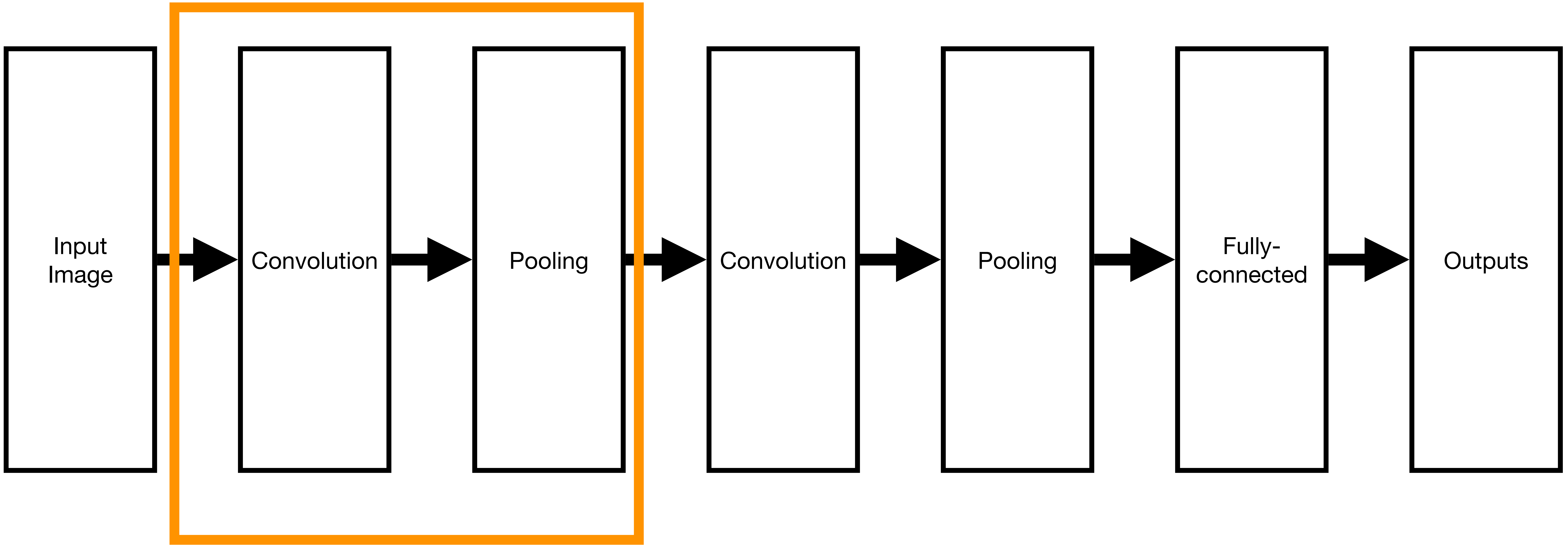


# Example: Translation Invariance




(Goodfellow 2016)

# Typical Architecture




Often convolution-then-pooling is collectively referred to as a "**convolution layer**"

# Summary

- Classifying images with a standard feedforward network requires vast quantities of **parameters** (and hence **data**)
- Convolutional networks add **pooling** and **convolution**
  - Sparse connectivity
  - Parameter sharing
  - Translation equivariance
- Fewer parameters means far more **efficient to train**