Policies and Value Functions

CMPUT 366: Intelligent Systems

S&B §3.5

Online Logistics

- **Flipped classroom model**
  1. Watch the **lecture video** (linked from eClass)
     - Perhaps you're doing that right now!
  2. Take the brief **comprehension quiz** (linked from eClass)
  3. Slides are available from eClass, as before
- **Assignments:** Same as before: download and submit using eClass
- **Final exam:** Online, details to be determined still

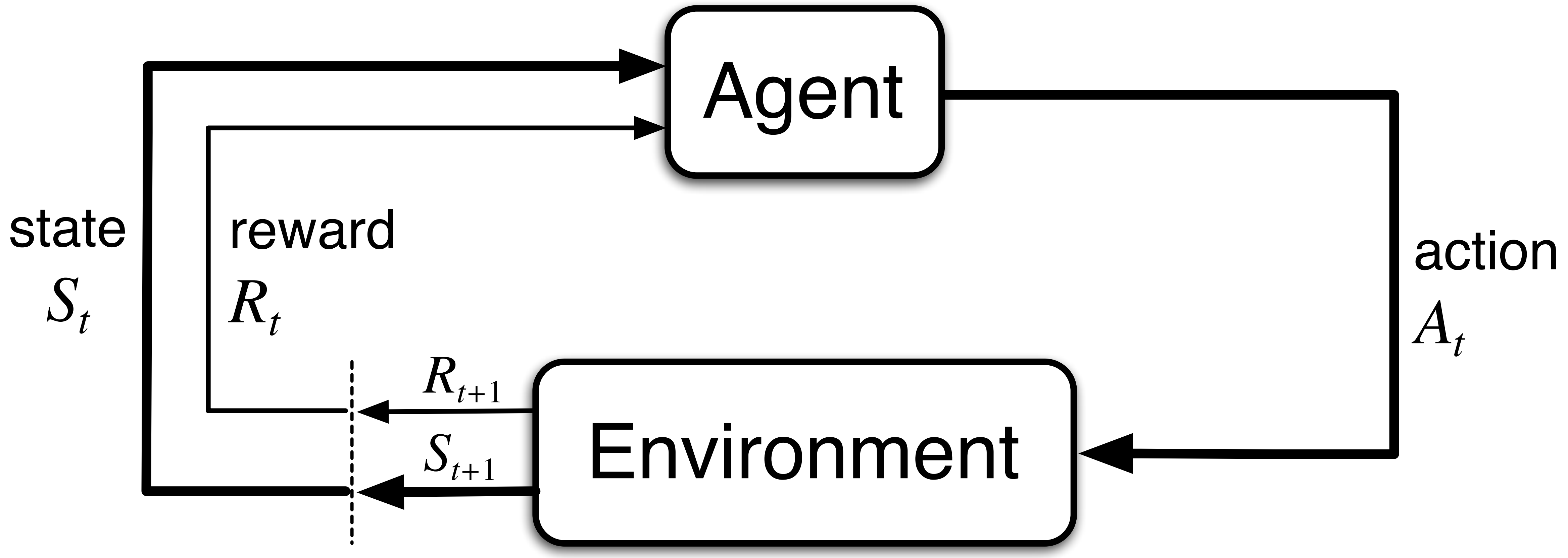
Lecture Outline

1. Recap & Logistics
2. Policies & Value Functions
3. Bellman Equations

Recap:
Interacting with the Environment

At each time $t = 1,2,3,\ldots$

1. Agent receives input denoting **current state** S_t
2. Agent chooses **action** A_t
3. Next time step, agent receives **reward** R_{t+1} and **new state** S_{t+1}, chosen according to a distribution $p(s', r \mid s, a)$



This interaction between agent and environment produces a **trajectory**:
$S_0, A_0, R_1, S_1, A_1, R_2, S_2, A_2, R_3, \ldots$

Policies

Question: How should an agent in a Markov decision process choose its **actions**?

- **Markov assumption**: The state incorporates all of the necessary information about the history up until this point
  - i.e., Probabilities of future rewards & transitions are the same from state S_t **regardless of how you got there**
- So the agent can choose its actions based **only** on S_t
- This is called a **policy**: $\pi(a \mid s) \in [0,1]$ is the probability of taking **action** a given that the **current state** is s

State-Value Function

- Once you know the **policy** π and the **dynamics** p, you can compute the probability of every possible state transition starting from any given state
- It is often valuable to know the **expected return** starting from a given state s under a given policy π (**why?**)
- The **state-value function** v_π returns this quantity:

$$v_\pi(s) \doteq \mathbb{E}_\pi[G_t \,|\, S_t = s]$$

$$= \mathbb{E}_\pi\left[\sum_{k=0}^{\infty} \gamma^k R_{t+k+1} \,\middle|\, S_t = s\right]$$

Action-Value Function

The **action-value function** $q_\pi(s, a)$ estimates the expected return G_t starting from state s if we

1. Take action a in state $S_t = s$, **and then**

2. Follow policy π for every state S_{t+1} afterward

$$
\begin{aligned}
q_\pi(s, a) &\doteq \mathbb{E}_\pi[G_t \,|\, S_t = s, A_t = a] \\
&= \mathbb{E}_\pi\left[\sum_{k=0}^{\infty} \gamma^k R_{t+k+1} \,\middle|\, S_t = s, A_t = a\right]
\end{aligned}
$$

Bellman Equations

$$\begin{aligned} G_t &\doteq R_{t+1} + \gamma R_{t+2} + \gamma^2 R_{t+3} + \dots \\ &= R_{t+1} + \gamma \left(R_{t+2} + \gamma R_{t+3} + \dots \right) \\ &= R_{t+1} + \gamma G_{t+1} \end{aligned}$$

Value functions satisfy a **recursive consistency condition** called the **Bellman equation:**

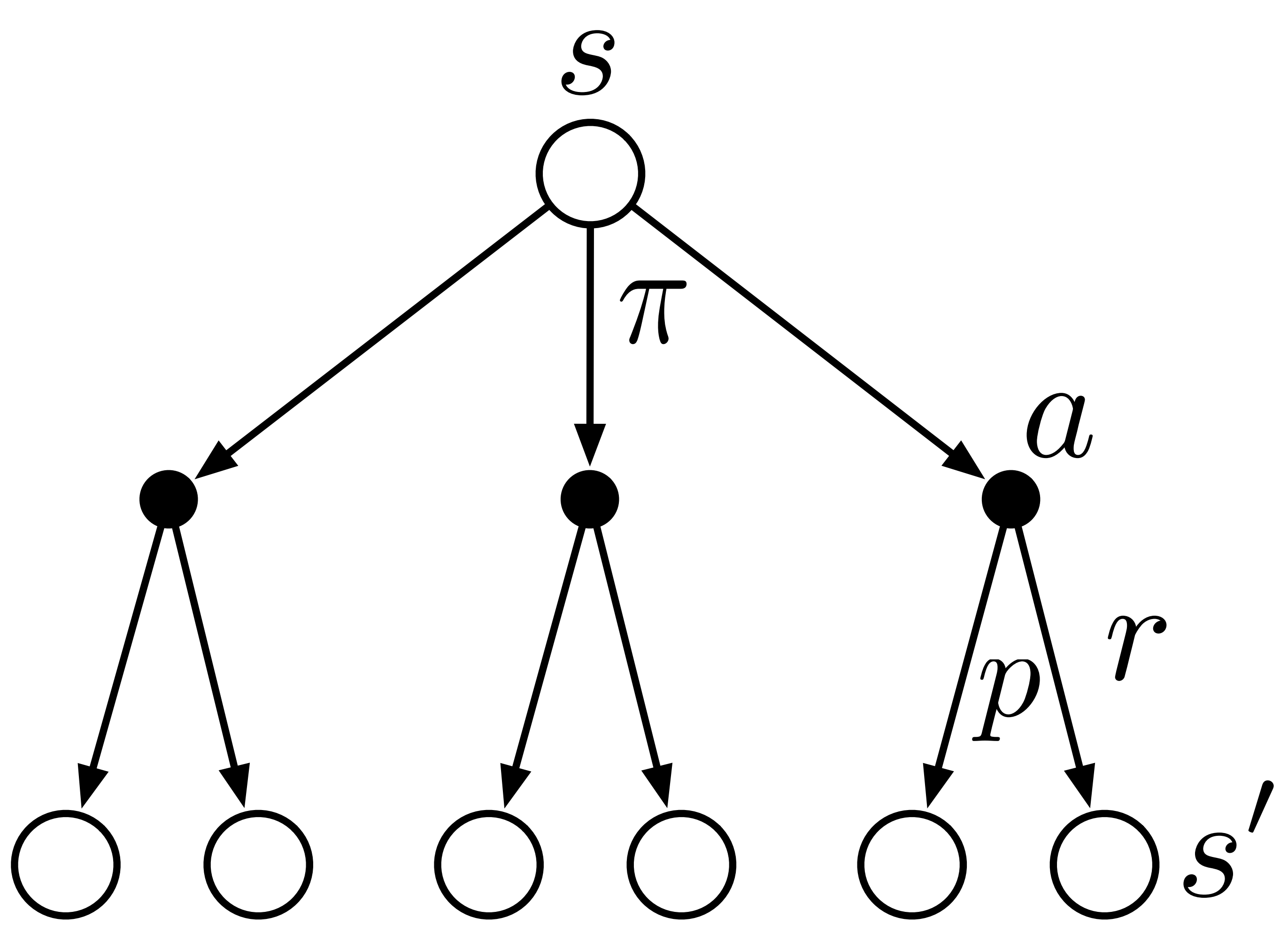
$$\begin{aligned} v_\pi(s) &\doteq \mathbb{E}_\pi[G_t \mid S_t = s] \\ &= \mathbb{E}_\pi[\boxed{R_{t+1} + \gamma G_{t+1}} \mid S_t = s] \\ &= \sum_a \pi(a \mid s) \sum_{s'} \sum_r p(s', r \mid s, a) \Big[r + \gamma \mathbb{E}_\pi[G_{t+1} \mid S_{t+1} = s'] \Big] \\ &= \sum_a \pi(a \mid s) \sum_{s', r} p(s', r \mid s, a) \Big[r + \gamma v_\pi(s') \Big] \end{aligned}$$

- v_π is the **unique solution** to π's Bellman equation
- There is also a Bellman equation for π's **action-value function**

Backup Diagrams

Backup diagrams help to visualize the flow of **information back to a state** from its successor states or action-state pairs:

$$
\begin{aligned}
v_\pi(s) &\doteq \mathbb{E}_\pi[G_t \mid S_t = s] \\
&= \sum_a \pi(a \mid s) \sum_{s',r} p(s', r \mid s, a)\Big[r + \gamma v_\pi(s')\Big]
\end{aligned}
$$

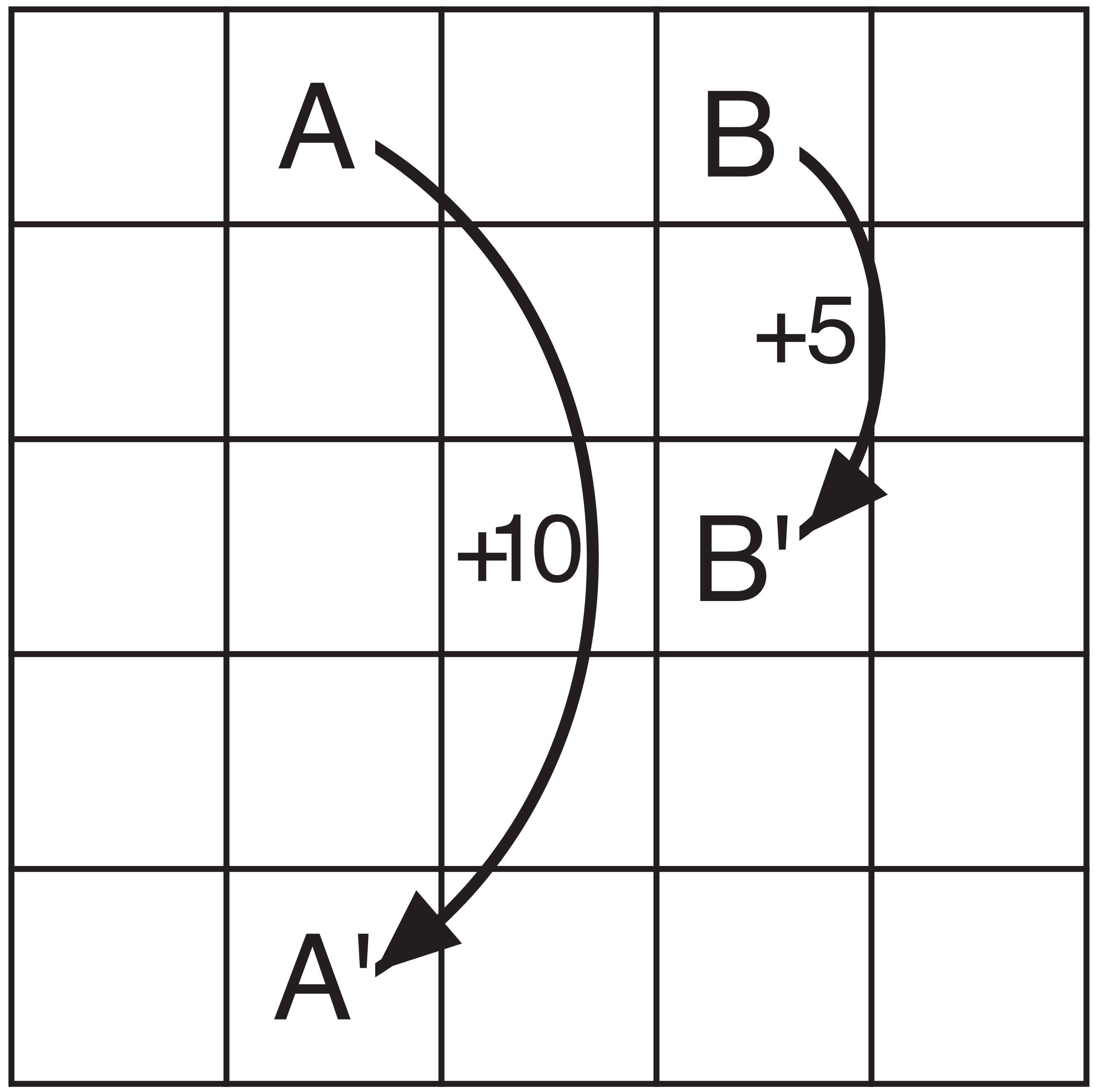


Backup diagram for v_π

(Image: Sutton & Barto, 2018)

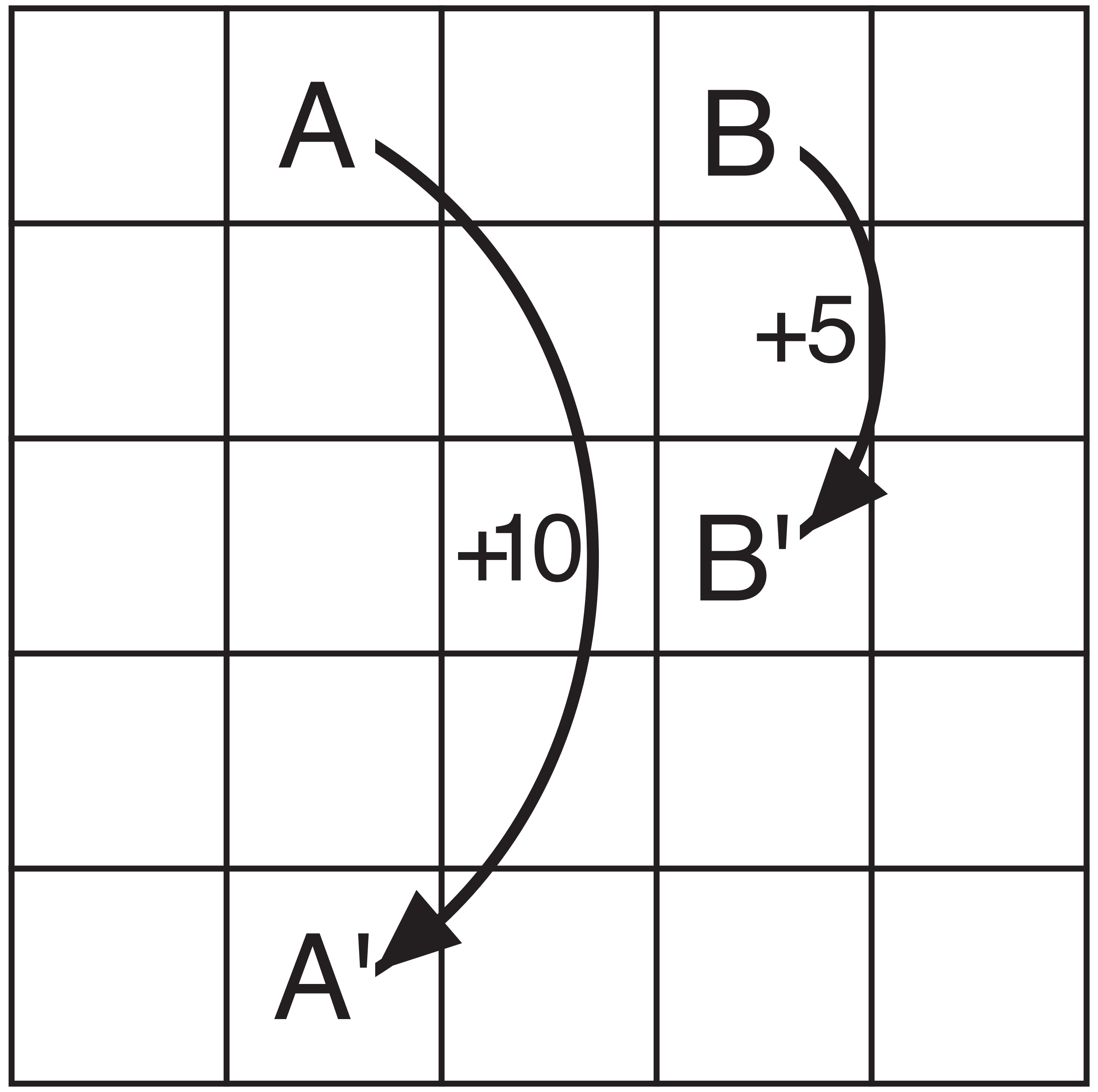
Return to GridWorld

- At each cell, can go `north`, `south`, `east`, `west`
- Try to go off the **edge**: reward of **-1**
- Leaving state **A**: takes you to state **A′**, reward of **+10**
- Leaving state **B**: takes you to state **B′**, reward of **+5**



(Image: Sutton & Barto, 2018)

Return to GridWorld



Reward dynamics

3.3	8.8	4.4	5.3	1.5
1.5	3.0	2.3	1.9	0.5
0.1	0.7	0.7	0.4	-0.4
-1.0	-0.4	-0.4	-0.6	-1.2
-1.9	-1.3	-1.2	-1.4	-2.0

State-value function v_π for random policy
$\pi(a|s) = 0.25$

(Image: Sutton & Barto, 2018)

Summary

- **Policies** map **states** to (distribution over) **actions**
- Given a **policy** π, every state s has an **expected value** $v_{\pi}(s)$
  - and every action a from state s has value $q_{\pi}(s, a)$
  - These are the **state-value** and **action-value** functions
- State-value and action-value functions satisfy the **Bellman equations**