Local Search

CMPUT 366: Intelligent Systems

P&M §4.7

Labs & Assignment #1

- **Assignment #1 will be released today**
  See eClass
- Due **February 3** before lecture
- This week's lab: **Thursday, 5:00pm to 8:00pm**, BS M 149 (this room!)
  - Not mandatory
  - You can get help from the TAs on your assignment in labs
  - Brief Python refresher at the beginning

Recap

- **Graph search problems** are an extremely general encoding for choosing a **sequence of actions** from a start state to a goal state
- Using **heuristic functions** can speed this process up
    - **A* search** is optimal but space-intensive
    - **Branch & bound depth-first search** is optimal and space efficient, but needs a good starting bound
- Varying the direction of search can exploit mismatches in forward and reverse **branching factors**

Lecture Outline

1. Recap & Logistics
2. Local Search
3. Hill Climbing
4. Randomized Algorithms

Searching for Goal Nodes

Sometimes, we know how to **recognize** a goal node, but not how to **construct** one.

Example (SAT problem): Given a Boolean formula,

$$P(X) = (X_1 \wedge X_2 \wedge \neg X_3) \vee \ldots \vee (\neg X_{k-2} \wedge \neg X_{k-1} \wedge X_k),$$

is there an assignment of truth values to the variables X_i that makes the formula true?

- **State** is the values of the different variables
- **Easy to recognize** when we've succeeded, but computing a "satisfying assignment" is **NP-complete** in general
- SAT is an example of a **constraint satisfaction problem**

Searching for Goal Nodes

We can encode SAT as a graph search problem (assignments as states, variable value changes as actions), *but*:

1. The **space is too big** to explore exhaustively
   - **Question:** How many states are there in a SAT problem with k variables?
   - Industrial SAT problems routinely have **hundreds of thousands** of variables
2. We don't care about the **sequence of actions**
   - Once we have a satisfying assignment, we are done

Local Search

- **Idea:** start from a random assignment, and then search around in the space of possible assignments
- Need not keep track of the sequence of moves that we took
- Intuitively:
    1. Select an assignment of a value to each variable
    2. Repeat:
        (i) Select a variable to change
        (ii) Select a new value for that variable
    3. until a satisfying assignment is found

Local Search Problem

Definition: Local Search Problem

- A **constraint satisfaction problem**: A set of **variables**, **domains** for the variables, and **constraints** on their joint assignment.
- **Neighbours function**: $neighbours(n)$
  - Maps from a node n to a set of "similar" nodes
- **Score function**: $score(n)$
  - Evaluates the "quality" of an assignment

Questions:

1. What are the **nodes**?
2. What are the **goal nodes**?

Neighbourhoods

- In previous graph search problems, the **successor function** represents states that can be **reached** from a given state by taking some actual action
  - In local search problems, the **neighbours function** is a **design decision**
  - We choose actions that will help us efficiently **explore the space** rather than trying to represent **actual actions**
- Usually the **neighbourhood** is states that differ in **small ways** from the current state (variable assignment)
  - E.g.: Assignments that differ in k different variables, possibly by a small amount
- **Question:** What might be a good **neighbourhood** function for **SAT**?

Heuristics vs. Scores

- Previously, the heuristic was **optional**, for improving efficiency
- In local search problems, the score function is **required**
    - The state space is **too big** to exhaustively explore, so uninformed search is not an option
    - Sometimes we don't even have a goal, we just want to **maximize the quality** of the state
- Example scores: number of unsatisfied clauses (in SAT); number of violated constraints (in CSP)

Generic Local Search Algorithm

Input: a constraint satisfaction problem; a *neighbours* function; a *score* function to maximize; a *stop_walk* criterion

current := random assignment of values to variables
incumbent := *current*
repeat
    if *incumbent* is a satisfying assignment:
        return *incumbent*
    if *stop_walk*():
        current := new random assignment of values to variables
    else:
        select a *current* from *neighbours*(*current*)
    if *score*(*current*) > score(incumbent):
        incumbent := current
until termination

Hill Climbing

- **Idea:** Select the neighbour with the **highest score**
  - This is called an **improving step**
  - If no improving steps available, **halt** and return *incumbent*
- We'll move toward the best solution once we are close enough
- This algorithm is called **hill climbing**:
  - It seeks the **highest point** on the scoring function's graph
  - It moves only **uphill** (i.e., it makes only improving steps)

Hill Climbing Algorithm

Input: a constraint satisfaction problem; a *neighbours* function; a *score* function

current := random assignment of values to variables
incumbent := *current*
repeat
    if *incumbent* is a satisfying assignment:
        return *incumbent*
    if *False*:
        current := new random assignment of values to variables
    else:
        current := *n* from *neighbours*(*current*) with maximum *score*(*n*)
    if *score*(*current*) > *score*(*incumbent*):
        incumbent := *current*
    else:
        return *incumbent*
until termination

Questions:

1. Is hill climbing **complete**?
2. Is hill climbing **optimal**?

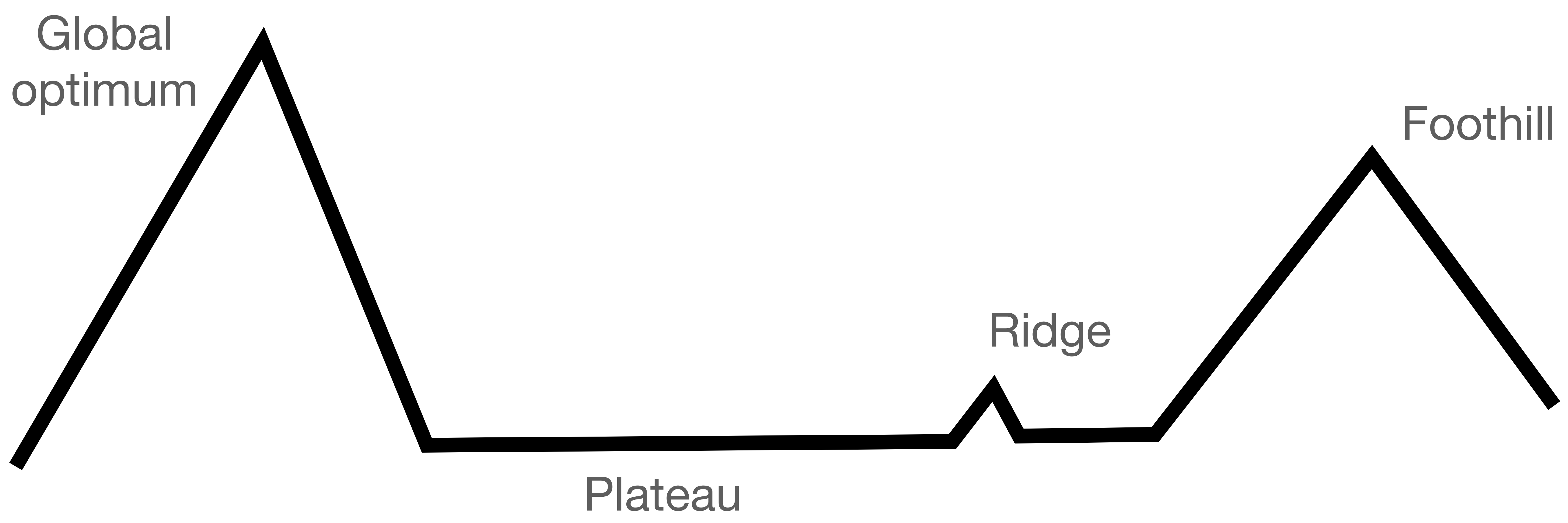
Hill Climbing Problems

1. **Foothills:** **Local maxima** that are not global maxima

2. **Plateaus:** Regions of the state space where the **score is uninformative**

3. **Ridges:** Foothills that would not be foothills with a **larger neighbourhood**

4. **Ignorance of the global optimum:** Unless we reach a satisfying assignment, we cannot be sure that an optimum returned by local search is the **global optimum**.



Randomized Algorithms

- Adding **random moves** can fix some hill climbing problems
- Two main kinds of random move:
    1. **Random restart:** Start searching from a **completely** random new location
    2. **Random step:** Choose a random **neighbour**
- **Stochastic random search**: Add both kinds of random moves to hill climbing

Stochastic Local Search

Input: a constraint satisfaction problem; a *neighbours* function; a *score* function to maximize; a *stop_walk* criterion; a *random_step* criterion

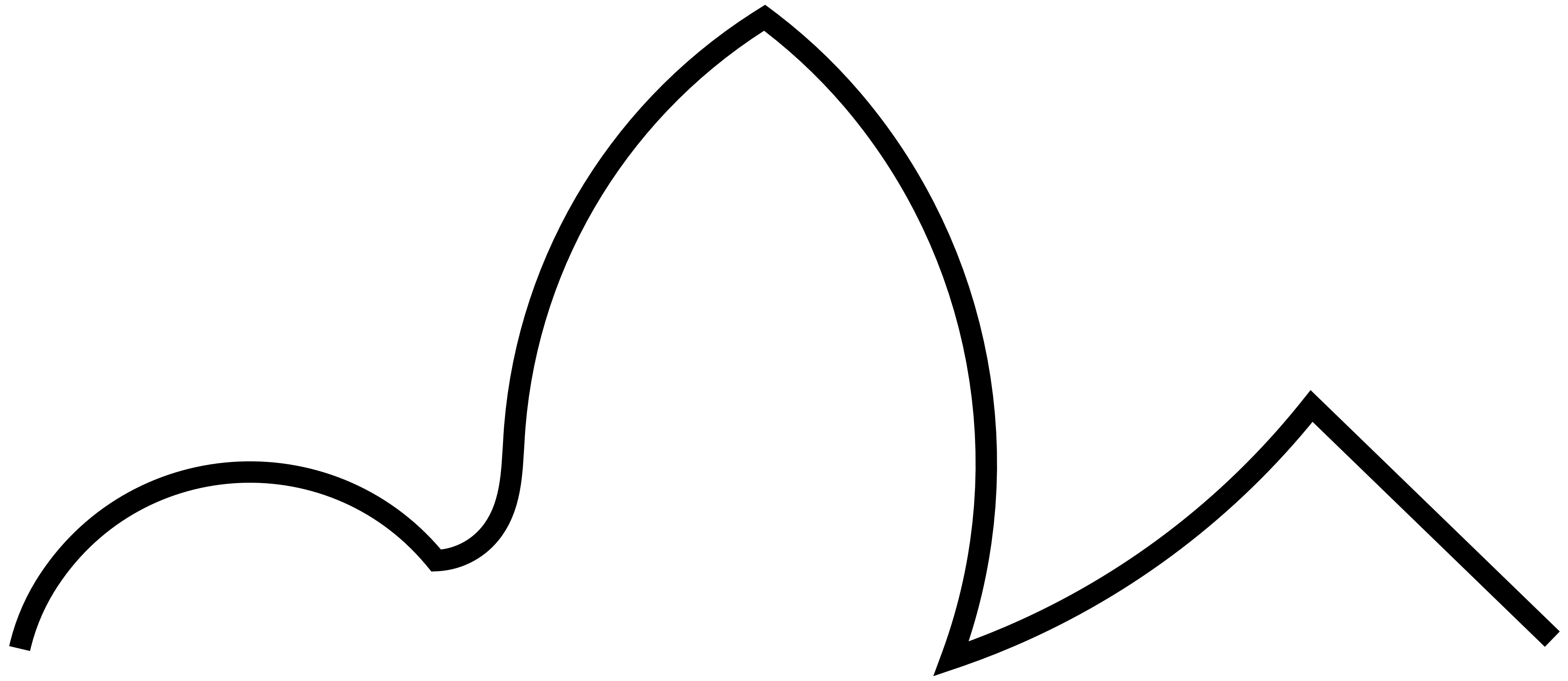
current := random assignment of values to variables
incumbent := *current*
repeat
    if *incumbent* is a satisfying assignment:
        return *incumbent*
    if *stop_walk*():
        current := new random assignment of values to variables
    else if *random_step*():
        current := a random element from *neighbours*(*current*)
    else:
        current := *n* from *neighbours*(*current*) with maximum *score*(*n*)
    if *score*(*current*) > score(incumbent):
        incumbent := current

Questions:

1. Is stochastic local search **complete**? (**Why?**)
2. Is stochastic local search **optimal**? (**Why?**)

Two Examples

- Consider two partial algorithms:
  1. Hill climbing plus **random restart**
  2. Hill climbing plus **random steps**
- **Question:** Which finds the maximum most easily on each of these two search spaces? Why?



Simulated Annealing

- **Idea: Start out** by searching pretty randomly, but become **more directed**
  - **Intuition:** Move to a good neighbourhood quickly, then search intensively in that neighbourhood
- Maintain a "**temperature**" T
- Choose new nodes more randomly at higher temperatures;
  Gradually decrease the temperature (according to a **cooling schedule**)
- At each step:
  1. Randomly choose a neighbour new
  2. **Always accept** (i.e., assign to $current$) if $score(new) > score(current)$
  3. Else, accept with **probability** $e^{[(score(new) - score(current))/T]}$

Summary

- For some problems, we only care about finding a **goal node**, not the actions we took to find it
- **Local search:** Look for goal states by iteratively moving from a **current state** to a **neighbouring state**
  - **Hill climbing:** Always move to the **highest-score** neighbour
  - **Random step:** Sometimes choose a **random** neighbour
  - **Random restart:** Sometimes start again from an **entirely random** state
  - **Simulated annealing:** Random moves start very **random**, become more **greedy** over time