Uninformed Search

CMPUT 366: Intelligent Systems

P&M §3.5

Logistics

- **NO LAB THIS WEEK**
- Assignment #1 released next week

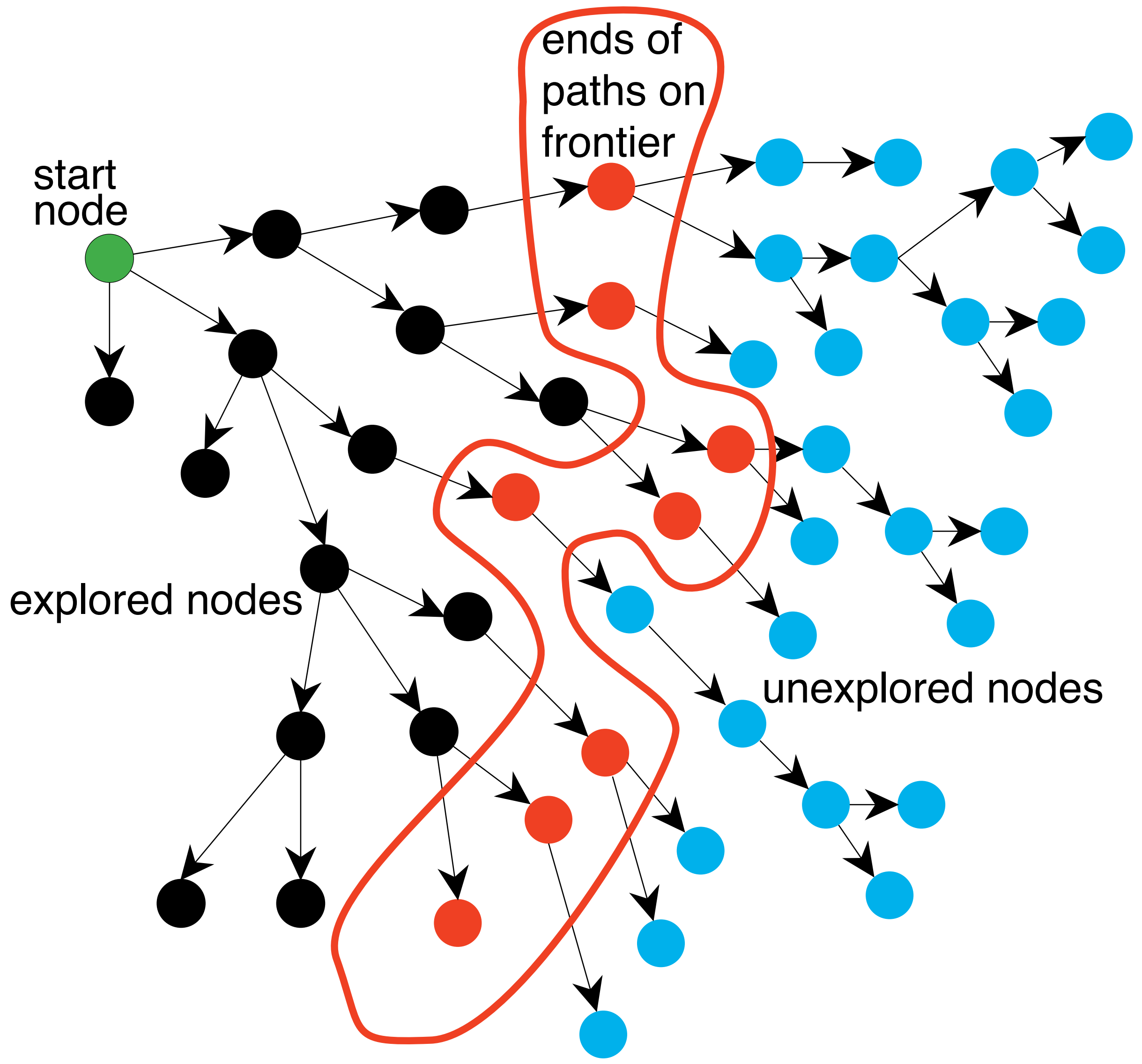
Recap: Graph Search

- Many AI tasks can be represented as **search problems**
  - A single generic **graph search algorithm** can then solve them all!
- A search problem consists of **states**, **actions**, **start states**, a **successor function**, a **goal** function, optionally a **cost** function
- **Solution quality** can be represented by labelling **arcs** of the search graph with **costs**

Recap: Generic Graph Search Algorithm

Input: a *graph*; a set of *start nodes*; a *goal* function

frontier := { <*s*> | *s* is a start node}
while *frontier* is not empty:
  select a path $<n_1, n_2, ..., n_k>$ from *frontier*
  remove $<n_1, n_2, ..., n_k>$ from *frontier*
  if *goal*(n_k):
    return $<n_1, n_2, ..., n_k>$
  for each neighbour *n* of n_k: (i.e., **expand** node n_k)
    add $<n_1, n_2, ..., n_k, n>$ to *frontier*
end while



https://artint.info/2e/html/ArtInt2e.Ch3.S4.html

- Which value is **selected** from the frontier defines the **search strategy**

Lecture Outline

1. Logistics & Recap
2. Properties of Algorithms and Search Graphs
3. Depth First Search
4. Breadth First Search
5. Iterative Deepening Search
6. Least Cost First Search

Algorithm Properties

What properties of algorithms do we want to analyze?

- A search algorithm is **complete** if it is guaranteed to find a solution within a finite amount of time whenever a solution exists.
- The **time complexity** of a search algorithm is a measure of how much **time** the algorithm will take to run, in the **worst case**.
    - In this section we measure by number of paths **added to the frontier**.
- The **space complexity** of a search algorithm is a measure of how much **space** the algorithm will use, in the **worst case**.
    - We measure by maximum number of paths **in the frontier**.

Search Graph Properties

What properties of the **search graph** do algorithmic properties depend on?

- **Forward branch factor**: Maximum number of neighbours
  Notation: b

- **Maximum path length**. (Could be infinite!)
  Notation: m

- Presence of **cycles**

- Length of the **shortest** path to a **goal** node

Depth First Search

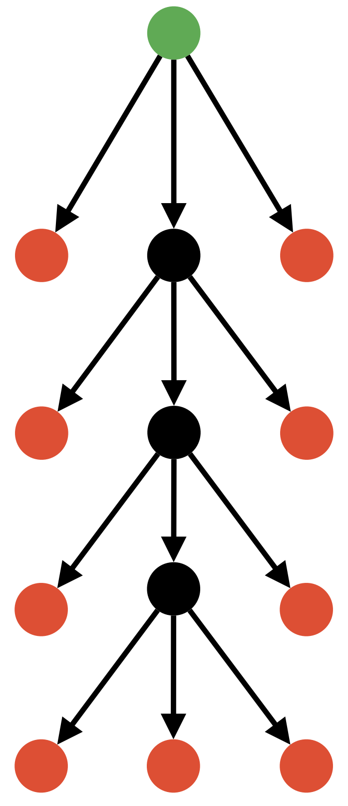
Input: a *graph*; a set of *start nodes*; a *goal* function

frontier := { <*s*> | *s* is a start node}
while *frontier* is not empty:
    select the newest path <n_1, n_2, ..., n_k> from *frontier*
    remove <n_1, n_2, ..., n_k> from *frontier*
    if *goal*(n_k):
        return <n_1, n_2, ..., n_k>
    for each neighbour *n* of n_k:
        add <n_1, n_2, ..., n_k, *n*> to *frontier*
end while

Question:

What **data structure** for the frontier implements this search strategy?

Depth First Search



Depth-first search always removes one of the **longest** paths from the frontier.

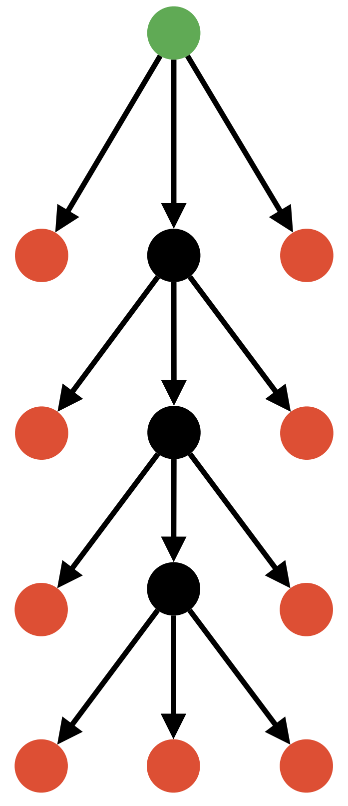
Example:
Frontier: $[p_1, p_2, p_3, p_4]$
successors(p_1) = $\{n_1, n_2, n_3\}$

What happens?

1. Remove p_1; test p_1 for goal
2. Add $\{<p_1,n_1>, <p_1,n_2>, <p_1,n_3>\}$ to **front** of frontier
3. New frontier: $[<p_1,n_1>, <p_1,n_2>, \boxed{<p_1,n_3>}, p_2, p_3, p_4]$
4. *p*2 is selected only after **all paths starting with p_1** have been explored

Question: When is $<p_1,n_3>$ selected?

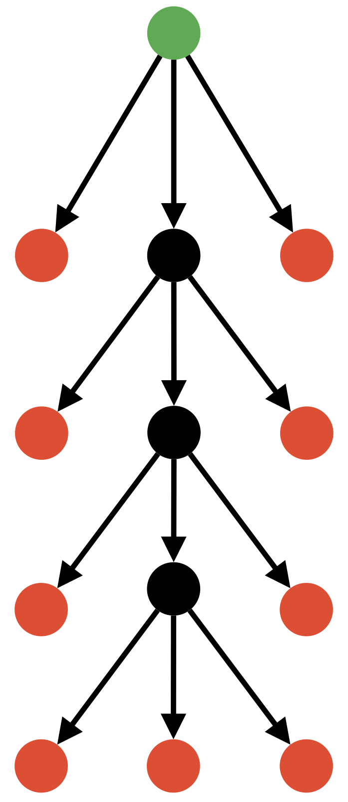
Depth First Search Analysis



For a search graph with maximum branch factor *b* and maximum path length *m*...

1. What is the worst-case **time complexity**?
   - [A: $O(m)$] [B: $O(mb)$] [C: $O(b^m)$] [D: it depends]
2. When is depth-first search **complete**?
3. What is the worst-case **space complexity**?
   - [A: $O(m)$] [B: $O(mb)$] [C: $O(b^m)$] [D: it depends]

When to Use Depth First Search



- When is depth-first search **appropriate**?
  - Memory is restricted
  - All solutions at same approximate depth
  - Order in which neighbours are searched can be tuned to find solution quickly
- When is depth-first search **inappropriate**?
  - Infinite paths exist
  - When there are likely to be shallow solutions
    - Especially if some other solutions are very deep

Breadth First Search

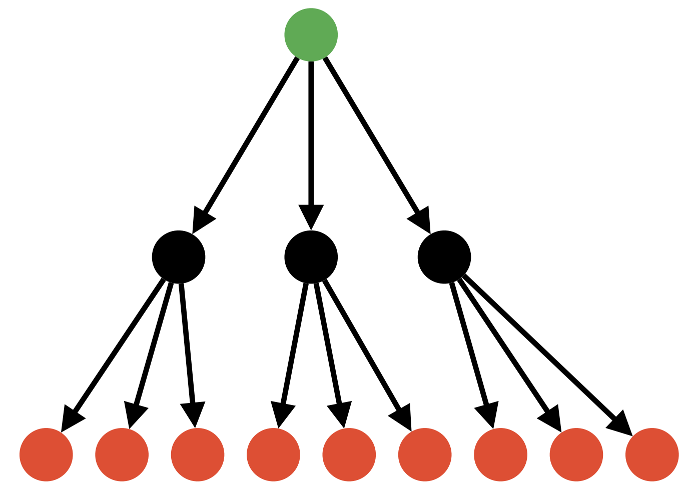
Input: a *graph*; a set of *start nodes*; a *goal* function

frontier := { <*s*> | *s* is a start node}
while *frontier* is not empty:
    select the oldest path $\langle n_1, n_2, \ldots, n_k \rangle$ from *frontier*
    remove $\langle n_1, n_2, \ldots, n_k \rangle$ from *frontier*
    if *goal*(n_k):
        return $\langle n_1, n_2, \ldots, n_k \rangle$
    for each neighbour n of n_k:
        add $\langle n_1, n_2, \ldots, n_k, n \rangle$ to *frontier*
end while

Question:

What **data structure** for the frontier implements this search strategy?

Breadth First Search



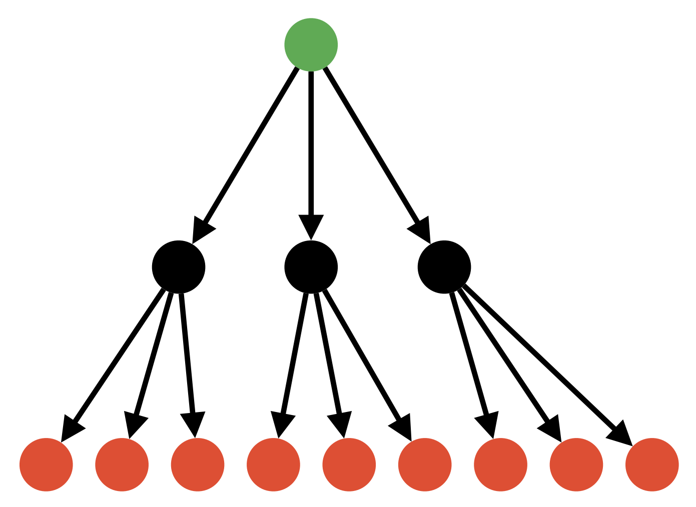
Breadth-first search always removes one of the **shortest** paths from the frontier.

Example:
Frontier: $[p_1, p_2, p_3, p_4]$
successors(p_1) = $\{n_1, n_2, n_3\}$

What happens?

1. Remove p_1; test p_1 for goal
2. Add $\{<p_1,n_1>, <p_1,n_2>, <p_1,n_3>\}$ to **end** of frontier:
3. New frontier: $[p_2, p_3, p_4, <p_1,n_1>, <p_1,n_2>, <p_1,n_3>,]$
4. p_2 is selected **next**

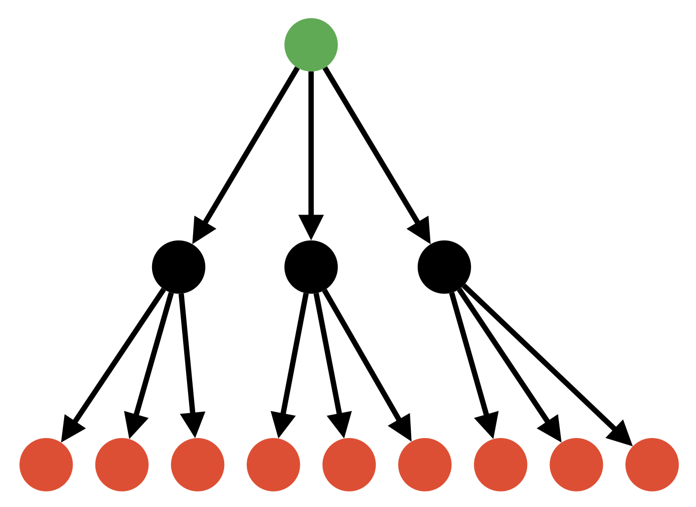
Breadth First Search Analysis



For a search graph with maximum branch factor b and maximum path length m...

1. What is the worst-case **time complexity**?
   - [A: $O(m)$] [B: $O(mb)$] [C: $O(b^m)$] [D: it depends]
2. When is breadth-first search **complete**?
3. What is the worst-case **space complexity**?
   - [A: $O(m)$] [B: $O(mb)$] [C: $O(b^m)$] [D: it depends]

When to Use
Breadth First Search



- When is breadth-first search **appropriate?**
  - When there might be infinite paths
  - When there are likely to be shallow solutions, *or*
  - When we want to guarantee a solution with fewest arcs
- When is breadth-first search **inappropriate?**
  - Large branching factor
  - All solutions located deep in the tree
  - Memory is restricted

Comparing DFS vs. BFS

	Depth-first	Breadth-first
Complete?	Only for finite graphs	Complete
Space complexity	$O(mb)$	$O(b^m)$
Time complexity	$O(b^m)$	$O(b^m)$

- Can we get the space benefits of depth-first search without giving up completeness?
- Run depth-first search to a maximum depth
  - then try again with a larger maximum
  - until either goal found or graph completely searched

Iterative Deepening Search

Input: a *graph*; a set of *start nodes*; a *goal* function

for *max_depth* from 1 to **∞**:
    Perform **depth-first search** to a maximum depth *max_depth*
end for

Iterative Deepening Search

Input: a *graph*; a set of *start nodes*; a *goal* function

```
more_nodes := True
while more_nodes:
    frontier := { <s> | s is a start node}
    for max_depth from 1 to ∞:
        more_nodes := False
        while frontier is not empty:
            select the newest path <n1, n2, ..., nk> from frontier
            remove <n1, n2, ..., nk> from frontier
            if goal(nk):
                return <n1, n2, ..., nk>
            if k < max_depth:
                for each neighbour n of nk:
                    add <n1, n2, ..., nk, n> to frontier
            else if nk has neighbours:
                more_nodes := True
```

Iterative Deepening Search Analysis

For a search graph with maximum branch factor b and maximum path length m...

1. What is the worst-case **time complexity**?
    - [A: $O(m)$] [B: $O(mb)$] [C: $O(b^m)$] [D: it depends]
2. When is iterative deepening search **complete**?
3. What is the worst-case **space complexity**?
    - [A: $O(m)$] [B: $O(mb)$] [C: $O(b^m)$] [D: it depends]

When to Use
Iterative Deepening Search

- When is iterative deepening search **appropriate**?
  - Memory is limited, and
  - Both deep and shallow solutions may exist
    - or we prefer shallow ones
  - Tree may contain infinite paths

Optimality

Definition:
An algorithm is **optimal** if it is guaranteed to return an optimal (i.e., **minimal-cost**) solution **first**.

Question: Which of the three algorithms presented so far is optimal? Why?

Least Cost First Search

- *None* of the algorithms described so far is guided by **arc costs**
    - BFS and IDS are implicitly guided by **path length**, which can be the same for uniform-cost arcs
- They return a path to a goal node as soon as they happen to blunder across one, but it may not be the optimal one
- **Least Cost First Search** is a search strategy that is **guided by arc costs**

Least Cost First Search

Input: a *graph*; a set of *start nodes*; a *goal* function

i.e., cost($<n_1, n_2, \ldots, n_k>$) ≤ cost(p) for all other paths $p \in$ *frontier*

frontier := { $<s>$ | s is a start node}
while *frontier* is not empty:
  select the cheapest path $<n_1, n_2, \ldots, n_k>$ from *frontier*
  remove $<n_1, n_2, \ldots, n_k>$ from *frontier*
  if *goal*(n_k):
    return $<n_1, n_2, \ldots, n_k>$
  for each neighbour n of n_k:
    add $<n_1, n_2, \ldots, n_k, n>$ to *frontier*
end while

Question:

What **data structure** for the frontier implements this search strategy?

Least Cost First Search Analysis

- Least Cost First Search is **complete** and **optimal** if there is $\varepsilon > 0$ with cost($<n_1,n_2>$) $> \varepsilon$ for every arc $<n_1,n_2>$:

    1. Suppose $<n_1,n_2,\ldots,n_k>$ is the optimal solution
    2. Suppose that p is any non-optimal solution
       So, cost(p) $>$ $<n_1,n_2,\ldots,n_k>$
    3. For every $1 \leq \ell \leq k$, cost($<n_1,n_2,\ldots,n_\ell>$) $<$ cost(p)
    4. So p will never be removed from the frontier before $<n_1,n_2,\ldots,n_k>$

- What is the worst-case **space complexity** of Least Cost First Search?
  [A: $O(m)$] [B: $O(mb)$] [C: $O(b^m)$] [D: it depends]
- When does Least Cost First Search have to expand **every node** of the graph?

Summary

- Different **search strategies** have different properties and behaviour
  - **Depth first search** is space-efficient but not always complete or time-efficient
  - **Breadth first search** is complete and always finds the shortest path to a goal, but is not space-efficient
  - **Iterative deepening search** can provide the benefits of both, at the expense of some time-efficiency
  - All three strategies must potentially expand **every node**, and are not guaranteed to return an **optimal solution**
- **Least cost first** is essentially breadth-first search with an optimality guarantee